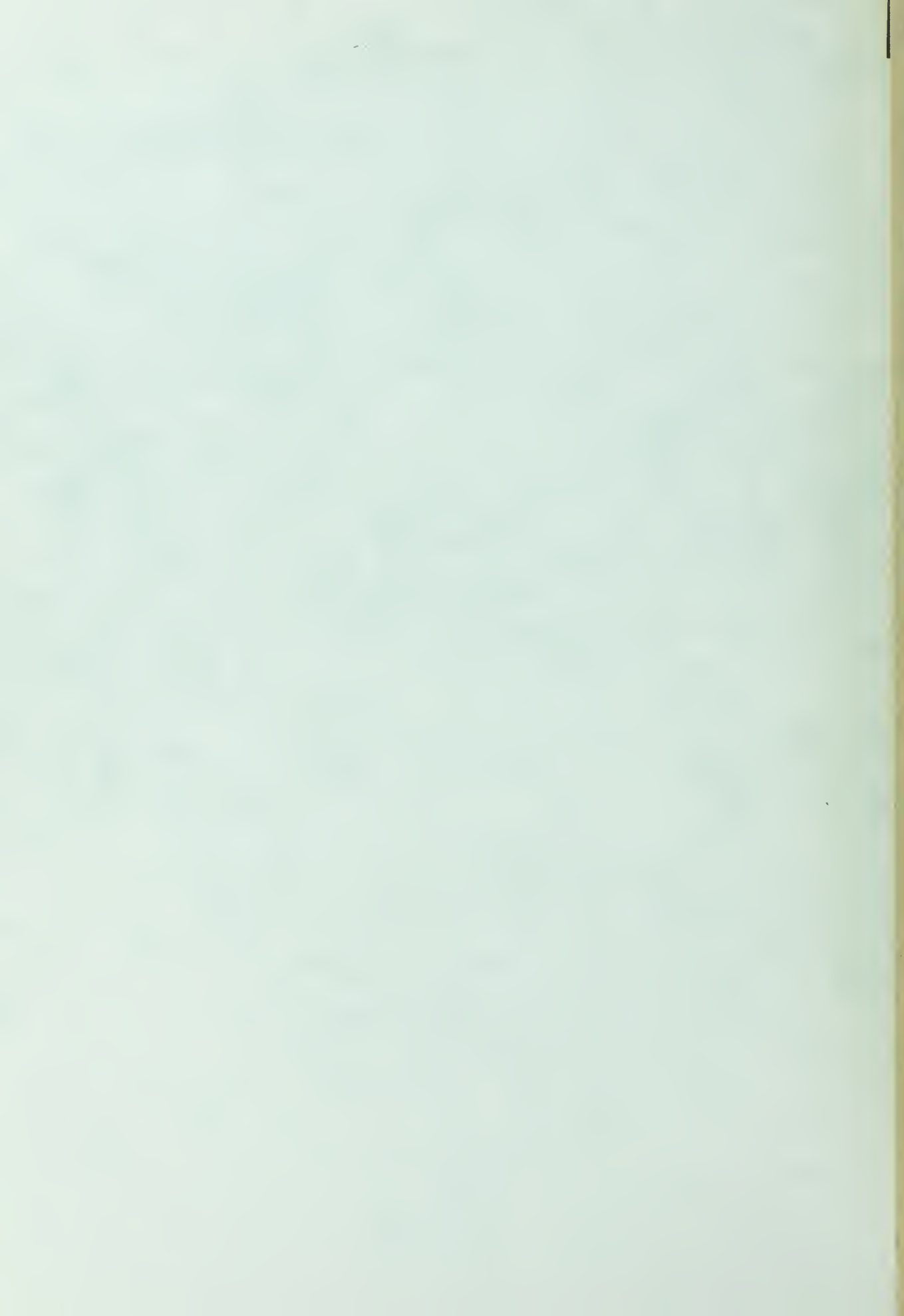


DYNAMICAL STABILITY AND MANEUVERABILITY  
OF  
DYNAMICALLY UNSTABLE SHIP

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# NAVAL POSTGRADUATE SCHOOL

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# THESIS

DYNAMICAL STABILITY AND MANEUVERABILITY  
OF  
DYNAMICALLY UNSTABLE SHIPS

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of  
Dynamically Unstable Ship

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## ABSTRACT

The factors affecting the stability and the maneuverability of a dynamically unstable ship were studied using the linear and non-linear equations of the motion of the ship. The DSL/360 Language was used to simulate the dynamics and to study both unstable motions and standard maneuvers.





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## TABLE OF SYMBOLS

$a_{eff}$	= effective aspect ratio
$A_f$	= area of movable fin
$A_R$	= area of rudder
$C_D$	= drag coefficient
$C_L$	= lift coefficient
$O$	= origin of reference axes fixed in the ship
$\mathcal{O}$	= midship section
$x, y, z$	= system of reference axes fixed in the ship
$x_o, y_o, z_o$	= system of reference axes fixed in the earth
$x_G, y_G, z_G$	= components of distance of C.G. of the ship along $x$ , $y$ and $z$ -axis, respectively.
$x_f$	= distance from $O$ to center of pressure of fin along $x$ -axis.
$k_1, k_2, k'$	= Lamb's coefficients (see ref. [1])
$k_1, k_2$	= coefficients of rudder deflection rate.
$I_x, I_y, I_z$	= mass moments of inertia of a ship about $x, y$ and $z$ -axis, respectively.
$m_2'$	= lateral added mass, non-dimensionalized.
$m_x'$	= actual plus added mass in the longitudinal direction non-dimensionalized.
$m$	= mass of ship
$m_x'$	= rotational added mass, non-dimensionalized.
$\phi$	= roll angle
$\theta$	= pitch angle
$\psi$	= yaw angle



$\beta$  = angle of attack in yaw on the hull (or drift angle).

$\delta$  = angular deflection of rudder

$U$  = resultant velocity of the fluid relative to a control surface or a fixed fin.

$p, q, r$  = components of resultant angular velocity of the ship about the  $x, y$  and  $z$ -axis, respectively  
(  $= \dot{\phi}, \dot{\theta}, \dot{\psi}$  ).

$u, v, w$ , = components of  $U$  along the  $x, y$  and  $z$ -axis, respectively.

$D$  = drag force

$F$  = total force

$L$  = lift force or length of ship (LBP)

$X, Y, Z$  = components of resultant total force acting at the origin  $O$  directed along  $x, y$  and  $z$ -axis, respectively.

$X_0, Y_0, Z_0$  = values of  $X, Y$  and  $Z$  at initial equilibrium condition.

$\vec{H}$  = angular momentum

$H$  = draft of ship

$\vec{\Omega}$  = angular velocity

$\vec{M}$  = resultant total moments

$K, M, N$  = components of a resultant total moment acting on a ship about the  $x, y$  and  $z$ -axis, respectively.

$Y_v$  = partial derivative of  $Y$  with respect to  $v$

$Y_v =$  " " "  $\dot{v}$

$Y_r =$  " " "  $r$

$Y_r =$  " " "  $\dot{r}$

$Y_\delta =$  " " "  $\delta$



$N_v$	=	partial derivative of N with respect to v
$N_v^\cdot$	=	" " $\dot{v}$
$N_r$	=	" " r
$N_r^\cdot$	=	" " $\dot{r}$
$N_\delta$	=	" " $\delta$

Note:

- 1) Symbol of prime ('), except where otherwise stated indicates the values of an item non-dimensionalized.
- 2) Symbol of dot (·) indicates differential value of an item with respect to time.
- 3) Subscript f indicates the value of an item of fin.
- 4) Subscript h indicates the value of an item of ship's hull.
- 5) Subscript  $f_1$  indicates the value of an item of the large deadwood included rudder.
- 6) Subscript  $f_2$  indicates the value of an item of the small deadwood or rudder only.





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## I. INTRODUCTION

A ship is said to be dynamically stable on a straight course or in a turn of constant curvature if, when slightly disturbed from it's steady motion, it will soon resume that same motion along a slightly shifted path without any correcting control being applied. A ship which is dynamically unstable in straight line motion cannot maintain straight line motion when there is no control. However, with regard to maneuverability, a ship which is too stable will not turn as tightly as a somewhat less stable ship so that a highly stable ship may compromise maneuverability. Therefore, a ship should be designed for a moderate amount of stability so as to be able to go straight without compromising maneuverability too much.

With these factors in mind then, the stability of five different ships was studied in Part III and maneuverability in Part IV. The first objective of this thesis was to investigate the stability of a ship with movable fin. The second objective was to investigate the maneuverability of an unstable ship in standard maneuvers to compare with those of a stable ship. A third objective was to apply DSL/360 Language to simulate the ship motions.



## II. EQUATION OF SHIP MOTION IN THE HORIZONTAL PLANE

As developed in Ref. 2, the general motion of a ship is that of a rigid body in six degrees of freedom subject to gravity and buoyancy forces as well as controlling forces and hydrodynamic reactions and subject to hydrodynamic or other disturbances or excitations.

Considering axes fixed to the body and parallel with the principal axes of inertia but with arbitrary origin as shown in Fig. 1, the linear and angular momentum equations may be written as

$$\begin{aligned}\vec{F} &= \frac{d}{dt}(m\vec{U}) = m\dot{\vec{U}} + \vec{\Omega} \times (m\vec{U}) \\ \vec{M} &= \frac{d}{dt}(\vec{H}) = \dot{\vec{H}} + \vec{\Omega} \times \vec{H}\end{aligned}\quad (1)$$

where

$m$  = mass of ship

$\vec{U}$  = linear velocity vector  
 $= u\vec{i} + v\vec{j} + w\vec{k}$

$u$  = rate of surging

$v$  = rate of swaying

$w$  = rate of heaving

$\vec{\Omega}$  = angular velocity vector

$= p\vec{i} + q\vec{j} + r\vec{k}$  (2)

$p$  = rate of roll =  $\phi$

$q$  = rate of yaw =  $\theta$

$r$  = rate of pitch =  $\psi$



$$\begin{aligned}\vec{F} &= \text{force vector acting on the body} \\ &= X\vec{i} + Y\vec{j} + Z\vec{k}\end{aligned}\quad (3)$$

$X$  = hydrodynamic force on body along x-axis

$Y$  = hydrodynamic force on body along y-axis

$Z$  = hydrodynamic force on body along z-axis

$\vec{h}$  = moment vector acting on body about a  
fixed point

$$= K\vec{i} + M\vec{j} + N\vec{k}\quad (4)$$

$K$  = rolling moment about x-axis

$M$  = pitching moment about the y-axis

$N$  = yawing moment about the z-axis

$H$  = angular momentum vector

$$= I_x p\vec{i} + I_y q\vec{j} + I_z r\vec{k}\quad (5)$$

$I_x, I_y, I_z$  = mass moment of inertia about the axis  
x, y and z respectively.

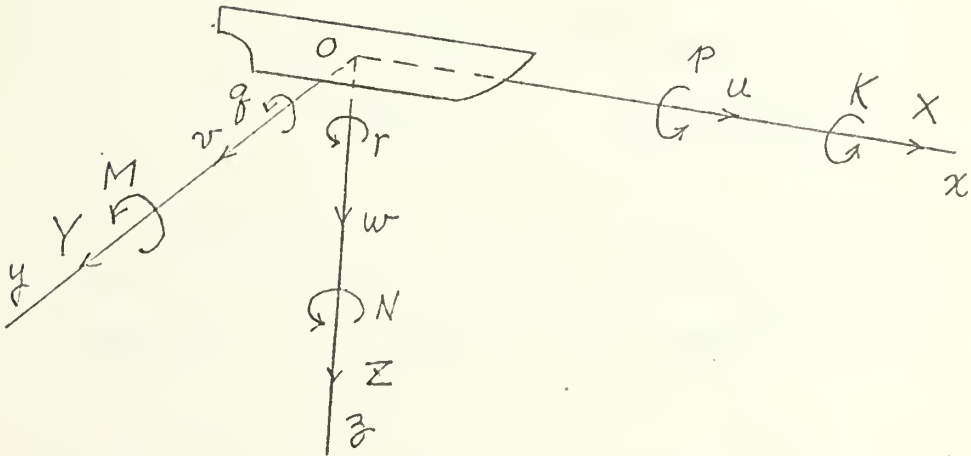


FIGURE 1. Body axes





Inserting Equations (2), (3), (4) and (5) in Eq. (1) gives:

$$\begin{aligned}
 X &= m \left\{ \dot{u} + q w - r v - \chi_q (q^2 + r^2) + y_q (p q - \dot{r}) + z_q (p r + \dot{q}) \right\} \\
 Y &= m \left\{ \dot{v} + r u - p w - y_q (r^2 + p^2) + z_q (q r - \dot{p}) + \chi_q (q p + \dot{r}) \right\} \\
 Z &= m \left\{ \dot{w} + p v - q u - z_q (p^2 + q^2) + \chi_q (r p - \dot{q}) + y_q (r q + \dot{p}) \right\} \\
 K &= I_x \dot{p} + (I_z - I_y) q r + m \left\{ y_q (\dot{w} + p v - q u) - z_q (\dot{v} + r u - p w) \right\} \\
 M &= I_y \dot{q} + (I_x - I_z) r p + m \left\{ z_q (\dot{u} + q w - r v) - \chi_q (\dot{w} + p v - q u) \right\} \\
 N &= I_z \dot{r} + (I_y - I_x) p q + m \left\{ \chi_q (\dot{v} + r u - p w) - y_q (\dot{u} + q w - r v) \right\}
 \end{aligned}
 \tag{6}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 K &= I_x \dot{p} + (I_z - I_y) q r + m \left\{ y_q (\dot{w} + p v - q u) - z_q (\dot{v} + r u - p w) \right\} \\
 M &= I_y \dot{q} + (I_x - I_z) r p + m \left\{ z_q (\dot{u} + q w - r v) - \chi_q (\dot{w} + p v - q u) \right\} \\
 N &= I_z \dot{r} + (I_y - I_x) p q + m \left\{ \chi_q (\dot{v} + r u - p w) - y_q (\dot{u} + q w - r v) \right\}
 \end{aligned} \right\} \tag{7}
 \end{aligned}$$

In Eq. (6) and (7), the left hand side represents the forces and moments acting on the ship and the right hand side represents the dynamic responses of the ship, respectively. The forces and moments acting on the ship depend on the geometry of the hull, the motion and the fluid. That is,

$$\vec{F} \text{ (or } \vec{m} \text{)} = f \text{ (properties of ship, properties of fluid, properties of ship motion)}$$

Since the properties of ship and fluid are constant for a given ship in a given fluid without excitation forces, the forces or moments may be expressed as a function of the motion as,

$$\vec{F} \text{ (or } \vec{m} \text{)} = f(\chi_0, y_0, z_0, \phi, \theta, \psi, u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}, \delta, \dot{\delta}, \ddot{\delta}, \text{ etc}) \tag{8}$$



where  $x_0$ ,  $y_0$  and  $z_0$  represent a system of reference axes fixed in the earth and

$\chi_0, \psi_0, \phi_0, \theta, \psi$  = orientation parameters

$\{u, v, w, p, q, r\}$   
 $\{\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}\}$  = motion parameters

$\delta, \dot{\delta}, \ddot{\delta}$  = control surface parameters

Considering motion in unrestricted calm water, only three degrees of freedom are of concern: yaw, sway and surge.

Therefore, the forces and moments are independent of the change in orientation with respect to the earth axes.

Furthermore since the forces and moments produced on the ship as a result of  $\dot{\delta}$  and  $\ddot{\delta}$  are normally negligible.

$$\vec{F}(\text{or } \vec{M}) = f(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \quad (9)$$

Since the motion is restrained to the horizontal plane for the surface ship in calm water as prescribed in the previous section, roll, pitching and heaving are taken as zero, i.e.,  $p=q=w=0$  in Eqs. (6) and (7). Furthermore, most ships have  $\psi_0 = 0$  so that Eqs. (6) and (7) may be written as,

$$\begin{aligned} X &= m(\dot{u} - rv - \chi_q r^2) \\ Y &= m(\dot{v} + ru + \chi_q \dot{r}) \\ N &= I_z \dot{r} + m\{\chi_q(\dot{v} + ur)\} \end{aligned} \quad (10)$$



From Eqs. (9) and (10) then, these become:

$$\begin{aligned}
 \text{X: } m(\dot{u} - rv - \chi_4 r^2) &= f_x(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \\
 \text{Y: } m(\dot{v} + ru + \chi_4 \dot{r}) &= f_y(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \\
 \text{N: } I_z \dot{r} + m\chi_4(\dot{v} + ur) &= f_N(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta)
 \end{aligned} \tag{11}$$



### III. DYNAMIC STABILITY OF THE SHIP

Dynamic stability is directly related to the magnitude of yaw and sway deviations caused by small initial disturbances. In many problems related to dynamic stability with small disturbances, the linear analysis of motion is quite useful and will be applied in the following to the present problem.

#### A. LINEAR EQUATION OF SHIP MOTION

To simplify the equation of ship motion, the following assumptions are made:

(1) The ship has adequate stability.

(2) The water is so calm that the rolling, pitching and heaving motions are negligible. Therefore, the motion of ship may be regarded as taking place in the horizontal plane only.

(3) Yaw and sway do not affect the forward speed appreciably, and small changes in forward speed do not affect yaw and sway motions so that the surge equation is decoupled.

(4) The ship has one rudder at the stern along the centerline and the effect of the propeller is disregarded.

(5) The forces and moments on hull and rudder are expressed by means of velocity and acceleration derivatives.

In order to linearize the equations of ship motion about an initial equilibrium condition of motion with the previous assumptions, only the linear terms in the change of the value





of the variables from equilibrium, i.e.  $\Delta u$ ,  $\Delta v$ ,  $\Delta r$ ,  $\Delta \dot{u}$ ,  $\Delta \dot{v}$ ,  $\Delta \dot{r}$  and  $\Delta \delta$  are considered.

From Eq. (9) then,

$$X = X_0 + X_u \Delta u + X_v \Delta v + X_r \Delta r + X_{\dot{u}} \Delta \dot{u} + X_{\dot{v}} \Delta \dot{v} + X_{\delta} \Delta \delta$$

Since in initial equilibrium  $v_0 = r_0 = \dot{u}_0 = \dot{v}_0 = \dot{r}_0 = \delta_0 = 0$ ,

$$\Delta v = v - v_0 = v$$

$$\Delta r = r - r_0 = r$$

$$\Delta \dot{u} = \dot{u} - \dot{u}_0 = \dot{u}$$

$$\Delta \dot{v} = \dot{v} - \dot{v}_0 = \dot{v}$$

$$\Delta \dot{r} = \dot{r} - \dot{r}_0 = \dot{r}$$

$$\Delta \delta = \delta - \delta_0 = \delta$$

Therefore,

$$X = X_0 + X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_{\delta} \delta \quad (12)$$

and likewise,

$$Y = Y_0 + Y_u \Delta u + Y_v v + Y_r r + Y_{\dot{u}} \dot{u} + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_{\delta} \delta \quad (13)$$

$$N = N_0 + N_u \Delta u + N_v v + N_r r + N_{\dot{u}} \dot{u} + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_{\delta} \delta$$

where  $X_0$ ,  $Y_0$  and  $N_0$  are the values of the initial equilibrium conditions.

In the initial equilibrium conditions of straight ahead motion at constant speed, there are no forces acting on the ship. Hence

$$X_0 = Y_0 = N_0 = 0$$



Furthermore, since the ship is symmetric about the x-z plane

$$Y_u = Y_{\dot{u}} = X_v = X_r = X_{\dot{r}} = N_u = N_{\dot{u}} = 0$$

and

$$X_{\delta} = 0$$

Therefore,

$$\begin{aligned} X &= X_u \Delta u + X_{\dot{u}} \dot{u} \\ Y &= Y_v v + Y_r r + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_{\delta} \delta \\ N &= N_v v + N_r r + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_{\delta} \delta \end{aligned} \quad (14)$$

Linearizing Eq. (10) gives

$$\begin{aligned} X &= m \left\{ (\dot{u}_0 + \Delta u) - (r_0 + \Delta r)(v_0 + \Delta v) - \chi_{\xi}(r_0 + \Delta r)^2 \right\} \\ &\doteq m(\Delta \dot{u} - \Delta r \Delta v - \chi_{\xi} \Delta r^2) \end{aligned}$$

This is simplified further by neglecting second order terms so that

$$X = m \Delta u = m \dot{u}$$

and likewise,

$$\begin{aligned} Y &= m(\dot{v} + r u_0 + \chi_{\xi} \dot{r}) \\ N &= I_z \dot{r} + m \chi_{\xi}(\dot{v} + r u_0) \end{aligned} \quad (15)$$

From Eqs. (14) and (15)

$$\begin{aligned} (X_{\dot{u}} - m) \dot{u} + X_u \Delta u &= 0 \\ (Y_{\dot{v}} - m) \dot{v} + Y_v v + (Y_{\dot{r}} - m \chi_{\xi}) \dot{r} + (Y_r - m u_0) r + Y_{\delta} \delta &= 0 \\ (N_{\dot{v}} - m \chi_{\xi}) \dot{v} + N_v v + (N_{\dot{r}} - I_z) \dot{r} + (N_r - m \chi_{\xi} u_0) r + N_{\delta} \delta &= 0 \end{aligned} \quad (16)$$



For convenience, Eq. (16) is non-dimensionalized. Every term of the first and second equations for the X-force and the Y-force in Eq. (16) has the dimensions of force and the third equation for moments has the dimension of a moment.

To non-dimensionalize these, the force equations are divided through by  $(\frac{\rho}{2} L H U^2)$  and the moment equation is divided by  $(\frac{\rho}{2} L^2 H U^2)$  where  $\rho$  = density of ship,  $L$  = ship's length,  $U$  = ship speed and  $H$  = draft of ship. This gives,

$$\begin{aligned} (X'_u - m') \dot{u}' + X'_u \Delta u &= 0 \\ (Y'_v - m') \dot{v}' + Y'_v v' + (Y'_r - m' \chi'_G) \dot{r}' + (Y'_r - m' u'_0) r' + Y'_\delta \delta & \\ (N'_v - m' \chi'_G) \dot{v}' + N'_v v' + (N'_r - I'_z) \dot{r}' + (N'_r - m' \chi'_G u'_0) r' + N'_\delta \delta & \end{aligned} \quad (17)$$

where

$$\begin{aligned} m' &= m / \frac{\rho}{2} L^2 H, \quad v' = v / U, \quad \dot{v}' = \dot{v} / \frac{U^2}{L} \\ u'_0 &= u_0 / U \approx 1 \quad \text{for small perturbations,} \\ I'_z &= I_z / \frac{\rho}{2} L^4 H, \quad r' = r / \frac{U}{L}, \quad \dot{r}' = \dot{r} L^2 / U^2 \\ \chi'_G &= \chi_G / L, \quad Y'_\delta = Y_\delta / \frac{\rho}{2} L H U^2, \quad N'_\delta = N_\delta / \frac{\rho}{2} L^2 H U^2, \\ Y'_v &= Y_v / \frac{\rho}{2} L H U, \quad Y'_r = Y_r / \frac{\rho}{2} L^2 H U, \quad N'_v = N_v / \frac{\rho}{2} L^2 H U, \\ N'_r &= N_r / \frac{\rho}{2} L^3 H U, \quad Y'_{\dot{v}} = Y_{\dot{v}} / \frac{\rho}{2} L^3 H, \quad N'_{\dot{v}} = N_{\dot{v}} / \frac{\rho}{2} L^3 H \\ N'_{\dot{r}} &= N_{\dot{r}} / \frac{\rho}{2} L^4 H \end{aligned}$$



In Eq. (17) the dimensionless time is defined as

$$t' = \frac{U}{L} t$$

## B. DYNAMIC STABILITY WITH FIXED CONTROL

Taking the Laplace Transform of Eq. (17) with the rudder fixed at  $\delta = 0$  gives:

$$\begin{aligned} [(\dot{Y}_v' - m')s + \dot{Y}_v']v'(s) + [(\dot{Y}_r' - m'\chi_{\xi}')s + (\dot{Y}_r' - m'u_o')]r'(s) &= 0 \\ [(\dot{N}_v' - m'\chi_{\xi}')s + \dot{N}_v']v'(s) + [(\dot{N}_r' - I_z')s + (\dot{N}_r' - m'\chi_{\xi}'u_o')]r'(s) &= 0 \end{aligned} \quad (18)$$

Then, using Cramer's rule

$$v'(s) = \frac{0}{\Delta}$$

$$r'(s) = \frac{0}{\Delta}$$

where

$$\Delta = \begin{vmatrix} (\dot{Y}_v' - m')s + \dot{Y}_v' & (\dot{Y}_r' - m'\chi_{\xi}')s + (\dot{Y}_r' - m') \\ (\dot{N}_v' - m'\chi_{\xi}')s + \dot{N}_v' & (\dot{N}_r' - I_z')s + (\dot{N}_r' - m'\chi_{\xi}') \end{vmatrix}$$

Expanding this determinant gives the characteristic equation

$$\Delta = A s^2 + B s + C \quad (19)$$





where

$$\begin{aligned}
 A &= (\gamma_v' - m')(\dot{N}_r' - I_z) - (\gamma_r' - m'\chi_q')(\dot{N}_v' - m'\chi_q') \\
 B &= (\gamma_v' - m')(\dot{N}_r' - m'\chi_q') - \gamma_v'(\dot{N}_r' - I_z) - \\
 &\quad \dot{N}_v'(\gamma_r' - m'\chi_q') - (\gamma_r' - m')(\dot{N}_v' - m'\chi_q') \\
 C &= \gamma_v'(\dot{N}_r' - m'\chi_q') - \dot{N}_v'(\gamma_r' - m')
 \end{aligned} \tag{20}$$

The roots of Eq. (19) are

$$\begin{aligned}
 \sigma_1 &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{-\frac{B}{A} + \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2} \\
 \sigma_2 &= \frac{-\frac{B}{A} - \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2}
 \end{aligned} \tag{21}$$

Therefore, the free responses of ship, i.e., sway and yaw motions are given by

$$\begin{aligned}
 v'(t') &= v_1 e^{\sigma_1 t'} + v_2 e^{\sigma_2 t'} \\
 r'(t') &= r_1 e^{\sigma_1 t'} + r_2 e^{\sigma_2 t'}
 \end{aligned} \tag{22}$$

where  $v_1$ ,  $v_2$ ,  $r_1$  and  $r_2$  are arbitrary constants depending on initial conditions.

As shown in Eq. (22), the dynamical stability depends on the roots of the characteristic equation. Since  $\sigma_1 \geq \sigma_2$ ,

$$\lim_{t' \rightarrow \infty} v'(t') = 0 \quad \text{and} \quad \lim_{t' \rightarrow \infty} r'(t') = 0$$



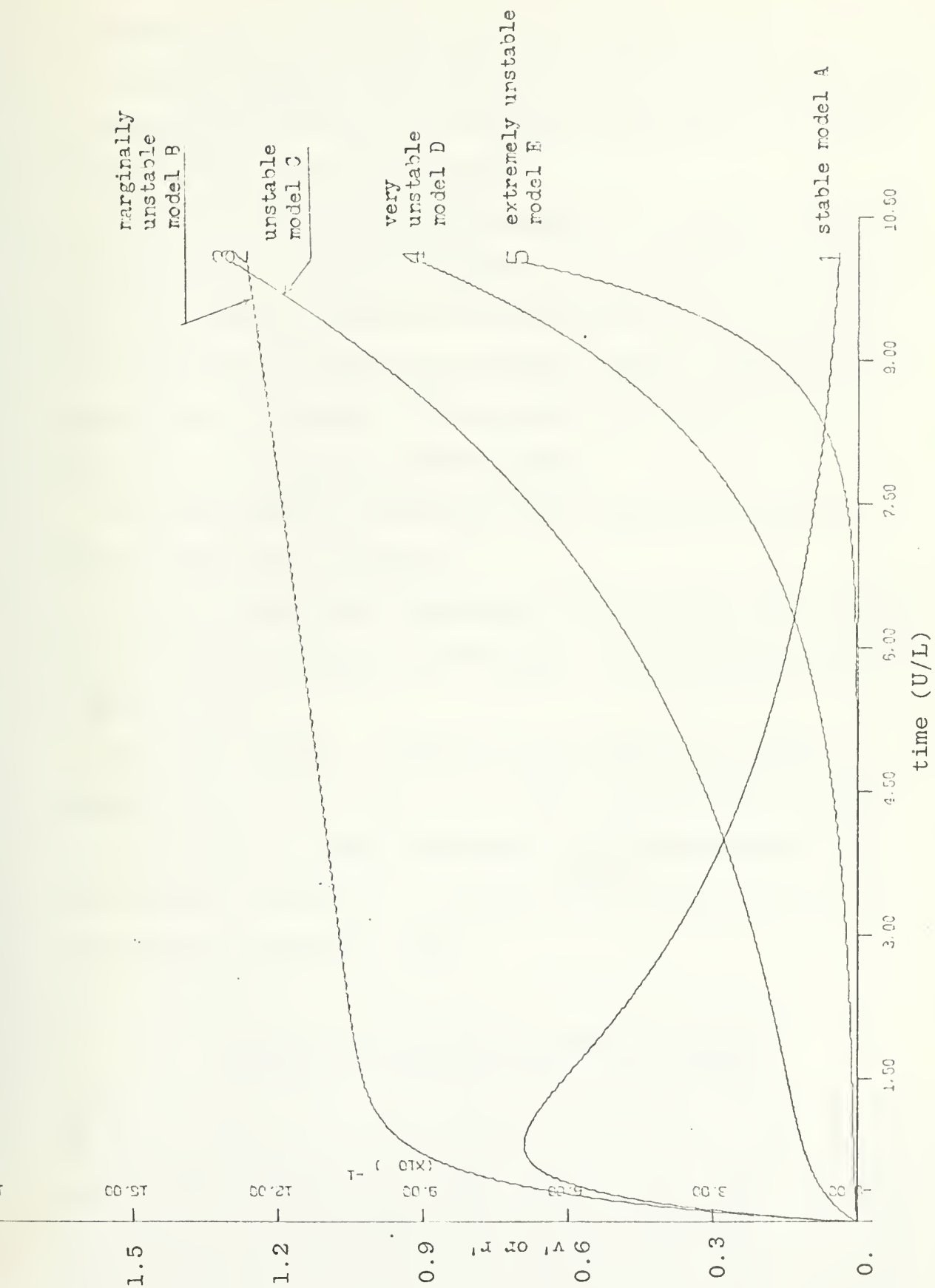


FIGURE 2. Response of the yaw rate and sway velocity to



if and only if  $\sigma_1 \leq 0$

If  $\sigma_1 > 0$  then,  $v'(\infty) \neq 0$  and  $r'(\infty) \neq 0$  which means that the ship is dynamically unstable in straight line motion. The root,  $\sigma_1$  is called the 'stability index'.

In Appendix B and computer program 1, a five foot Series 60 model of the Davidson Laboratory with block coefficient,  $C_B = 0.7$ , and no propeller, was modified to be (a) stable (b) marginally unstable (c) unstable (d) very unstable and (e) extremely unstable. These models are referred to as model A, B, C, D and E, respectively.

The hydrodynamic derivatives and stability indices of these models are in Table 1.

Eqs. (21) and (22) were simulated to obtain the response of each model to small a disturbance. (See computer program 7 and Fig. 2)

As shown in Fig. 2 (which is the result of computer program 7), the yaw rate,  $r$ , as well as the sway velocity,  $v$ , of the stable model A decreases as time increases but the unstable models B, C, D and E tend to diverge due to the positive stability index.

TABLE I. HYDRODYNAMIC DERIVATIVES AND STABILITY INDICES OF THE FIVE MODELS.

mod- el	$Y_v'$	$N_v'$	$Y_r'$	$N_r'$	$Y_{\dot{r}}'$	$N_{\dot{r}}'$	$Y_{\dot{v}}'$	$N_{\dot{v}}'$	$\sigma_1$
A	-0.3544	-0.0938	+0.0774	-0.0635	-0.1880	+0.0040	+0.0040	-0.0132	-0.3213
B	-0.3131	-0.1145	+0.0567	-0.0532	-0.1868	+0.0034	+0.0034	-0.0129	+0.0238
C	-0.3131	-0.1266	+0.0446	-0.0435	-0.1868	+0.0024	+0.0024	-0.0121	+0.2561
D	-0.3131	-0.1447	+0.0265	-0.0380	-0.1868	+0.0010	+0.0010	-0.0117	+0.5194
E	-0.1867	-0.1777	-0.0065	-0.0216	-0.1800	0.	0.	-0.0112	+1.3438



### C. STABILIZATION OF THE DYNAMICALLY UNSTABLE SHIP WITH FIXED RUDDER

As described in the previous section, a dynamically unstable ship has

$$\sigma_1 = \frac{-\frac{B}{A} + \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2} > 0$$

To stabilize this unstable ship,  $\sigma_1$  must be made less than or equal to zero, i.e.,  $\sigma_1 \leq 0$  or,

$$\sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})} \leq \frac{B}{A}$$

By the nature of hydrodynamic derivatives for the ship  $A > 0$  and  $B > 0$  always. Therefore,  $C$  must be greater than or equal to zero for stability. Thus,  $C$  is called the "stability criterion."

From Eq. (20)

$$C = Y_v'(N_r' - m' \chi_q') - N_v'(Y_r' - m') \geq 0 \quad (20a)$$

Then, considering the characteristics of each of the hydrodynamic derivative, mass and the distance of the C.G. from the origin of the fixed coordinates on the ship.

$Y_v$  is always large negative quantity

$N_r$  is always large negative quantity

$N_v$  may be positive or negative

$Y_r$  is small but may be positive or negative





$m$  is always large and positive

$X_G$  is always small but may be positive or negative

Finally,  $N_v$  has the greatest effect on dynamical stability since the term  $(Y_r' - m')$  is a large negative quantity in equation (20a). If, for example,  $N_v$  becomes more positive, the stability criterion  $C$  will become more positive so that the stability of the ship will be improved. It is noted that  $N_v$  can be controlled by a lifting surface.

#### 1. Sensitivity of Dynamic Stability to a Lifting Surface

Equation (21) was simulated using the computer program DSL/360 Language for the effect of the location, size and aspect ratio of a lifting surface (movable fin) on the models A, B, C, D and E. (See the computer program 2). The results are summarized in Table II and III.

The following are indicated by Tables II and III and by Figs. 3, 4 and 5:

1) The position of the fin at which the stability index is maximum is independent of the area and aspect ratio of the fin, because this position is the pivot point at which the relative flow is parallel to the centerline of the ship.

(See Fig. 14)

The parallel flow makes no attack angle on the fin so that  $Y_v$ ,  $N_v$ ,  $Y_r$  and  $N_r$  of the fin are zero but  $Y_v^*$ ,  $N_v^*$ ,  $Y_r^*$  and  $N_r^*$  are not zero as described in Appendix A. Therefore, the stability criterion  $C$  in Eq. (20) is the same as that of the



TABLE II. EFFECT OF FIN AREA AND FIN LOCATION WITH  
FIXED ASPECT RATIO ON DYNAMICAL STABILITY

Fin	$X_f/L$	A	B	C	D	E
No fin		-0.3213	+0.0238	+0.2561	+0.5194	+1.3438
$a_{eff}=1.5$	- 0.5	- .4366	- .1066	+ .1108	+ .3625	+1.1684
$A_f=0.0056$	$\mathcal{A}$	- .3492	- .0020	- .2324	+ .4968	+1.3223
$h_f=0.015$	pivot	- .32079	+ .0242	+ .2569	+ .5197	+1.3441
	point	(0.48)	(0.4)	(0.34)	(1.305)	(0.27)
	+ 0.5	- .3208	+ .0226	+ .2512	+ .5109	+1.3288
$a_{eff}=1.5$	- 0.5	- .5707	- .2616	- .0642	+ .1710	+ .9474
$A_f=.01265$	$\mathcal{A}$	- .3834	- .0341	+ .2026	+ .4682	+1.2944
$h_f=.0225$	pivot	- .3203	+ .0246	+ .2567	+ .5199	+1.3442
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3204	+ .0212	+ .2458	+ .5011	+1.3110
$a_{eff}=1.5$	- 0.5	- .7376	- .4583	- .2887	- .0780	- .6717
$A_f=.0225$	$\mathcal{A}$	- .4299	- .0780	+ .1617	+ .4286	+1.2548
$h_f=.03$	pivot	- .3197	+ .0250	+ .2571	+ .5202	+1.3444
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3199	+ .0196	+ .2393	+ .4893	+1.2888
$a_{eff}=1.5$	- 0.5	-1.0956	- .8930	- .7921	- .6486	- .0573
$A_f=0.0506$	$\mathcal{A}$	- .5532	- .1963	+ .0507	- .3198	+1.1427
$h_f=0.045$	pivot	- .3185	+ .0260	+ .2579	+ .5209	+1.3448
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3188	+ .0162	+ .2259	+ .4643	+1.2393
destabilizing area		0.415L~ 0.5L	0.35L~ 0.445L	0.3L~ 0.375L	0.27L~ 0.34L	0.24L~ 0.3L

Note: All values in column A,B,C,D and E (except the values in ( ) which mean pivot point distance from  $\mathcal{A}$ ) are  $G_1$  of model A,B,C,D and E, respectively.



TABLE III. EFFECT OF FIN LOCATION AND ASPECT RATIO WITH  
FIXED AREA ON DYNAMICAL STABILITY

Fin Type	$X_f/L$	Model A	B	C	D	E
No fin		-0.3213	+0.0238	+0.2561	+0.5194	+1.3438
$a_{eff} = 1.5$	- 0.5	-0.4366	-0.1066	+0.1108	+0.3625	+1.1684
$A_f=0.0056$	$\mathcal{X}$	-0.3492	-0.0020	+0.2324	+0.4968	+1.3223
$h_f=0.015$	pivot	-0.3208	+0.0242	+0.2569	+0.5197	+1.3441
	point	(0.48)	(0.40)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3208	+0.0226	+0.2512	+0.5109	+1.3288
$a_{eff} = 2.$	- 0.5	-0.4557	-0.1275	+0.0872	+0.3373	+1.1442
$A_f=0.0056$	$\mathcal{X}$	-0.3539	-0.0063	+0.2285	+0.4932	+1.3189
$h_f=0.0173$	pivot	-0.3207	+0.0242	+0.2565	+0.5197	+1.3439
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	+0.3208	+0.0224	+0.2505	+0.0510	+1.3265
$a_{eff} = 3.$	- 0.5	-0.4823	-0.1572	+0.0543	+0.3021	+1.1036
$A_f=0.0056$	$\mathcal{X}$	-0.3605	-0.0122	+0.2231	+0.4881	+1.3164
$h_f=0.0212$	pivot	-0.3206	+0.0242	+0.2565	+0.5197	+1.3440
	point	(0.48)	(0.40)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3207	+0.0221	+0.2495	+0.5078	+1.3235
$a_{eff} = 4.$	- 0.5	-0.5000	-0.1769	+0.0325	+0.2788	+1.0786
$A_f=0.0056$	$\mathcal{X}$	-0.3649	-0.0162	+0.2195	+0.4847	+1.3112
$h_f=0.02448$	pivot	-0.3206	+0.0244	+0.2565	+0.5198	+1.3441
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3206	+0.0219	+0.2488	+0.5066	+1.3215

Note: All values in column A,B,C,D and E (except the values in ( ) which mean pivot point distance from  $\mathcal{X}$ ) are  $\sigma_1$  of model A,B,C,D and E, respectively.



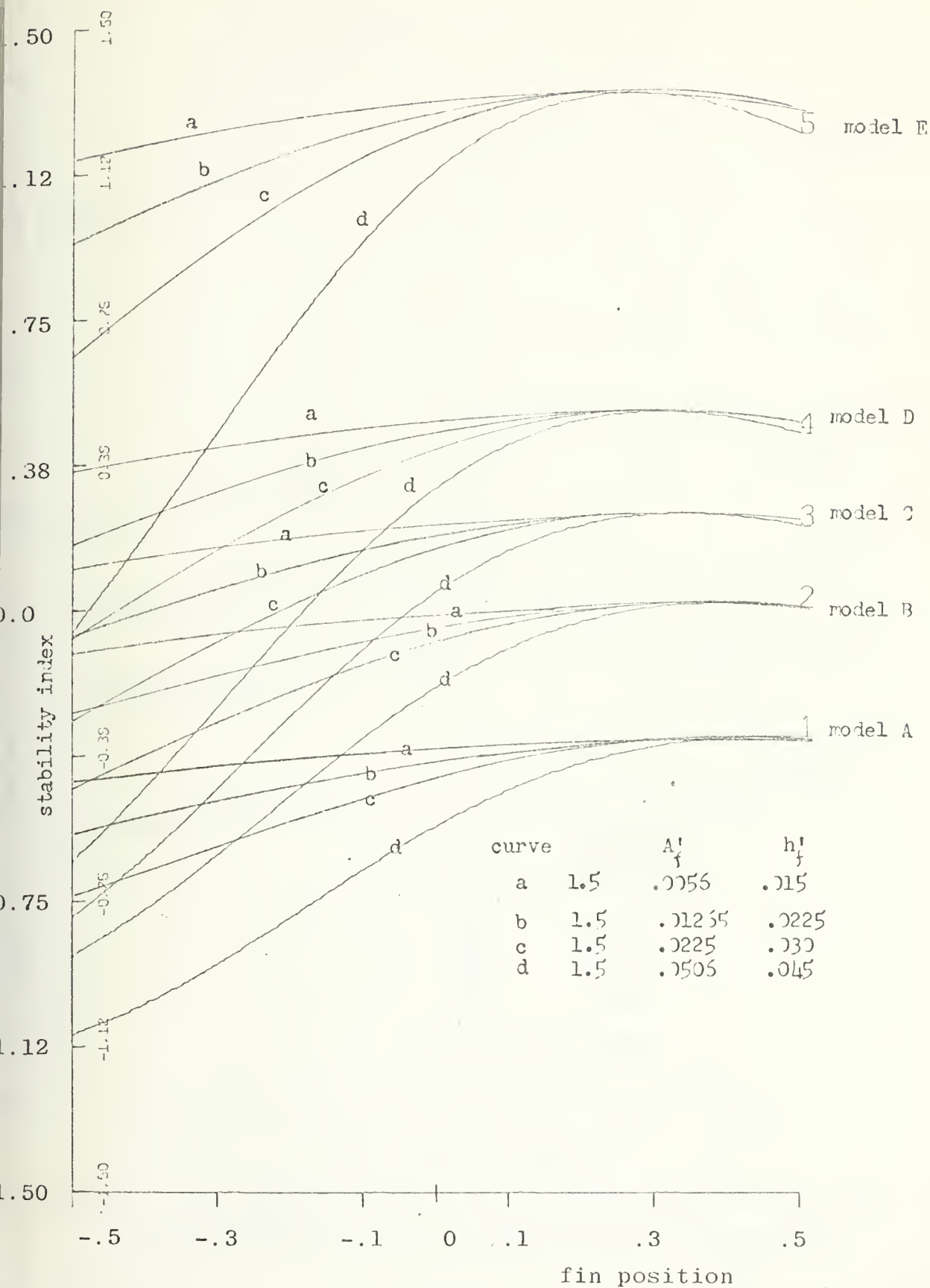


Fig. 3 The effect of fin location and fin area with constant aspect ratio on stability of the ship.





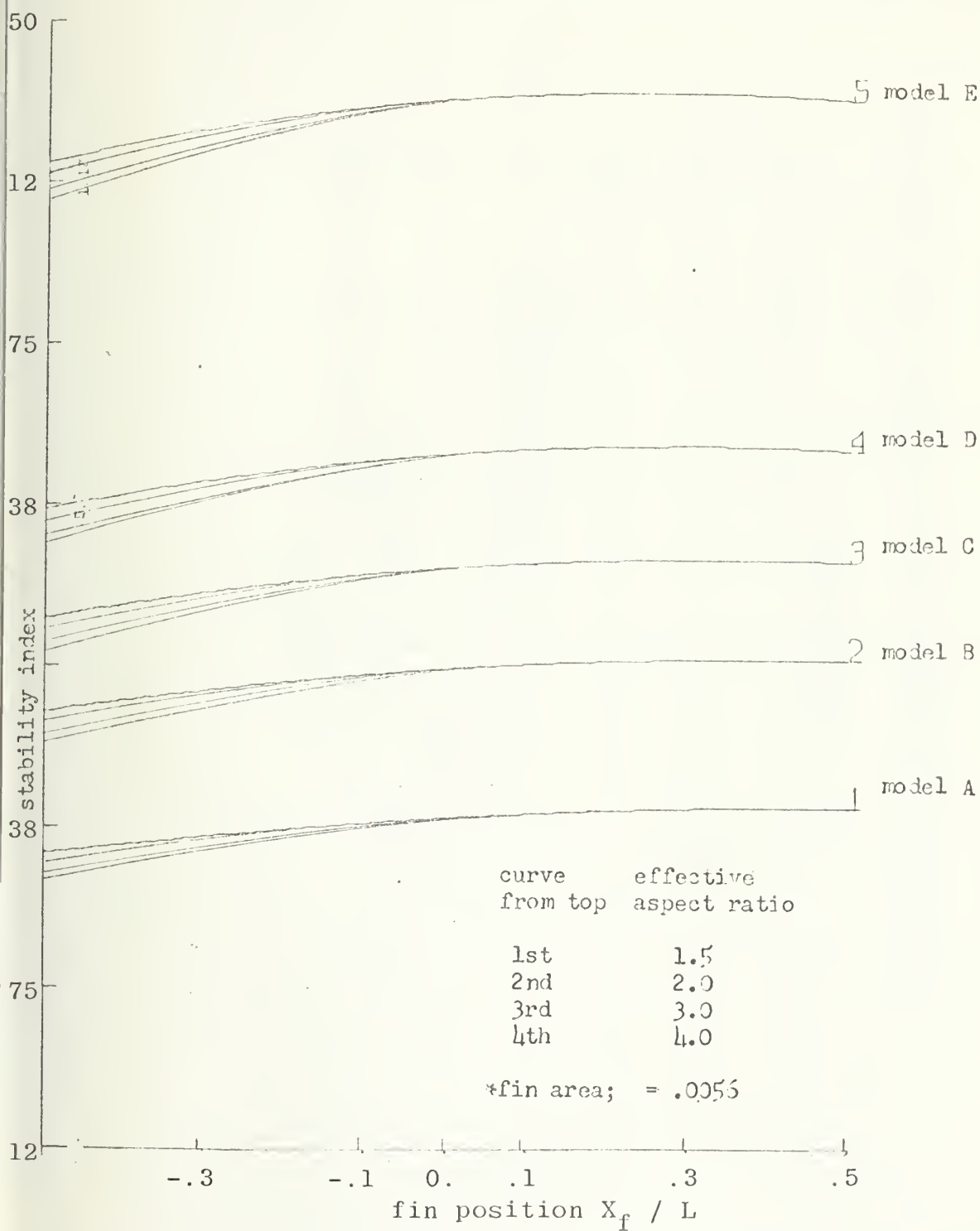


FIGURE 4. The effect of fin location and aspect ratio with constant area of fin on the stability of the ship.



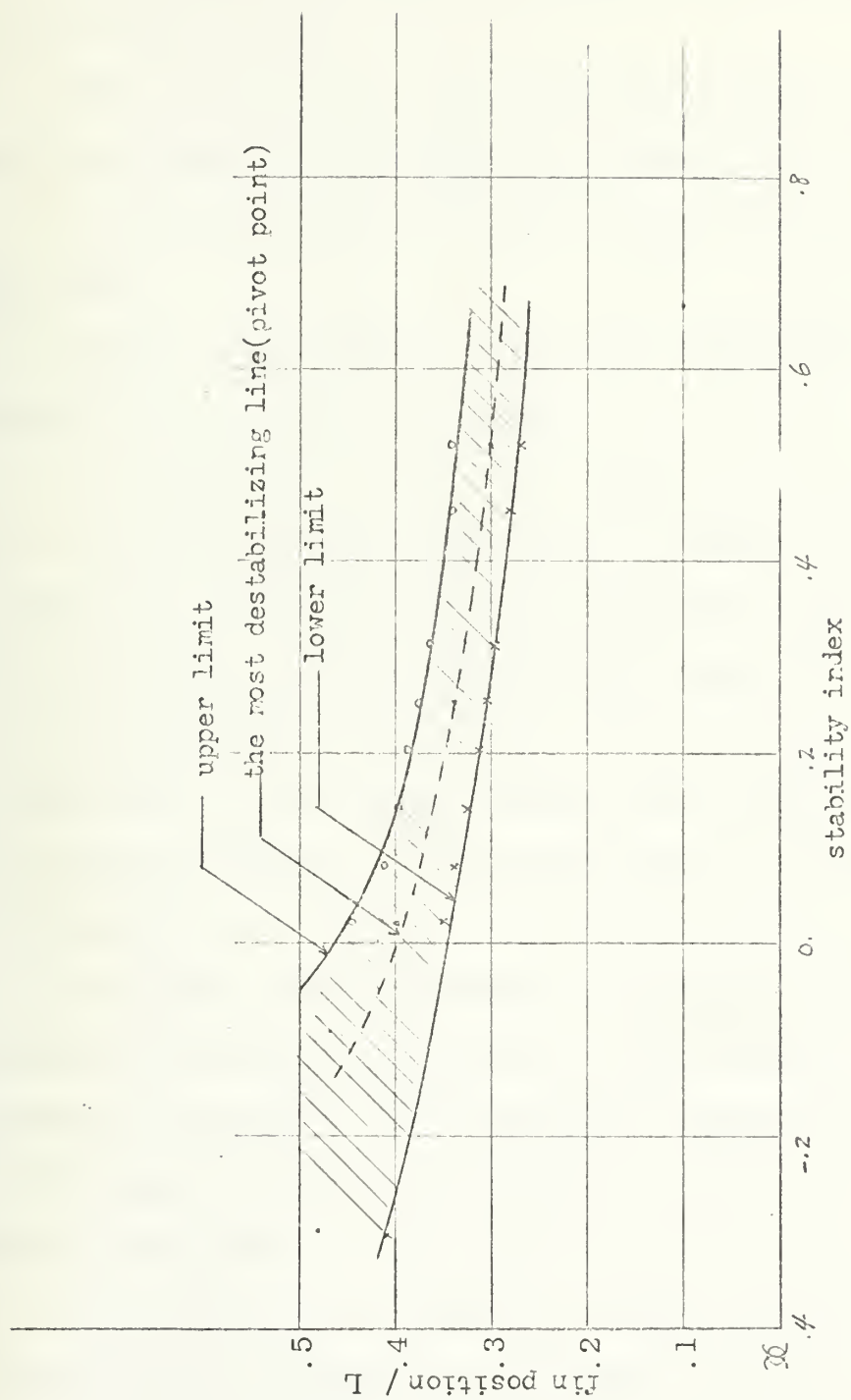


Fig.5 Destabilizing area of the fin and the most destabilizing point



no-fin condition. In other words, the stability criterion,  $C$  is minimum at the pivot point during shifting of the fin from stern to bow.

This pivot point is the position of the fin at which it exerts maximum destabilizing influences on the ship.

2) The effect of the fin on stability increases with fin size.

3) The more unstable the ship, the closer the pivot point moves toward the midship.

4) Compared to the no-fin condition, the placement of the fin near the pivot point has a negative effect on stability. This criterion is called the "destabilizing area" in this thesis. This destabilizing area is due to the relative change in  $A$ ,  $B$ , and  $C$  in Eq. (21) as the fin location is shifted.  $C$  is almost constant near the pivot point and the destabilizing area is independent of the size and type of the fin.

5) Moving the fin toward the stern from midship improves the dynamic stability of the unstable ship. Conversely, moving the fin forward of midship does not cause much change from no-fin condition regardless of the size or type of the fin.

6) Stability is sensitive to the fin area but rather insensitive to changes in the aspect ratio of the fin.

## 2. Sensitivity of Stability of the Unstable Ship to the Position of the C.G. of the Ship

It is clear that moving the C.G. toward the bow tends to make ' $C$ ' more positive in Eq. (20). The effect of moving



C.G. was studied by use of Eqs. (20) and (21) using DSL/360 Language for model C. The results are shown in Table IV. (See computer program 3).

TABLE IV. THE EFFECT OF LOCATION OF C.G. ON THE STABILITY INDEX

$X_{G/L}$	-0.4	-0.25	$\mathcal{X}$	+0.05	+0.1	+0.25
$\sigma_1$	+ .6424	+ .4022	+ .0775	+ .015	- .0473	- .2297

As shown in the above table, stability was improved by moving the C.G. toward the bow as expected. However, this is not a practical method of stabilizing an unstable ship.

#### D. STABILIZATION OF THE DYNAMICALLY UNSTABLE SHIP WITH RUDDER CONTROL

In previous sections, the stabilization of a dynamically unstable ship using lifting surface was discussed. However, it is clear that the dynamically unstable ship can be made directionally stable if the rudder can be forced to compensate for disturbances so that the ship will maintain course with only small oscillations in yaw and sway. If the deflection of the rudder,  $\delta$ , is proportional to the heading error,  $\psi$ , and the angular velocity,  $r = \dot{\psi}$ , then the equation of rudder deflection becomes

$$\delta(t') = k_1 \psi(t') + k_2 \dot{\psi}(t') \quad (23)$$





where  $k_1$  and  $k_2$  are constants of proportionality of the control system.

Inserting Eq. (23) into Eq. (17) and taking Laplace Transform gives

$$\begin{aligned} & \left[ (Y_v' - m')s + Y_v' \right] V(s) + \left[ (Y_r' - m'x_q')s^2 + (Y_r' - m' + Y_g'k_2)s + Y_g'k_1 \right] \psi(s) = 0 \\ & \left[ (N_v' - m'x_q')s + N_v' \right] V(s) + \left[ (N_r' - I_z')s^2 + (N_r' - m'x_q' + k_2N_g')s + k_1N_g' \right] \psi(s) = 0 \end{aligned} \quad (24)$$

As these algebraic equations of (24) are homogeneous, a non-trivial solution ( $v \neq \psi \neq 0$ ) demands the determinant of the system, i.e., characteristic equation, to be zero.

Let the characteristic equation be

$$As^3 + Bs^2 + Cs + D = 0 \quad (25)$$

where

$$\begin{aligned} A &= (Y_v' - m')(N_r' - I_z') - (Y_r' - m'x_q')(N_v' - m'x_q') \\ B &= (Y_v' - m')(N_r' - m'x_q' + k_2N_g') + Y_v'(N_r' - I_z') - \\ & \quad N_v'(Y_r' - m'x_q') - (Y_r' - m' + k_2Y_g')(N_v' - m'x_q') \\ C &= (Y_v' - m')(k_1N_g') + Y_v'(N_r' - m'x_q' + k_2N_g') - \\ & \quad N_v'(Y_r' - m' + k_2Y_g') - k_1Y_g'(N_v' - m'x_q') \\ D &= k_1Y_v'N_g' - k_1N_v'Y_g' \end{aligned}$$



Let the roots of Eq. (25) be  $\sigma_1', \sigma_2'$  and  $\sigma_3'$ . Then,

$$\begin{aligned} \psi(t') &= \psi_1 e^{\sigma_1' t'} + \psi_2 e^{\sigma_2' t'} + \psi_3 e^{\sigma_3' t'} \\ v'(t') &= v_1 e^{\sigma_1' t'} + v_2 e^{\sigma_2' t'} + v_3 e^{\sigma_3' t'} \end{aligned} \quad (26)$$

If and only if  $\sigma_1', \sigma_2'$  and  $\sigma_3'$  are negative, then  $\psi(\infty)$  and  $v(\infty)$  will be zero and the unstable ship will be stable.

Furthermore, if the roots are three repeated roots, i.e.,

$\sigma_1' = \sigma_2' = \sigma_3' < 0$ , the ship will be stable with the shortest possible response.

By designing for the optimum value of  $k_1$  and  $k_2$  in the control mechanism, the ship will be automatically controlled for suitable stability.

To find the three repeated negative roots of the characteristic Eq. (25), the 'MITROVIC METHOD' was applied. (See Appendix C).

In computer program 4, the optimum  $k_1$  and  $k_2$  for the three repeated negative roots of Eq. (25) were calculated for model C and Eq. (26) was simulated by the DSL/360 Language. (See computer program 4).

As shown in Fig. 5-1 which is the result of computer program 4, the response with proper  $k_1$  and  $k_2$  for three negative real roots of characteristic equation (25) is an exponential transient. Conversely, if the auto-steering device has some  $k_1$  and  $k_2$  which give complex roots of characteristic equation, the response will be stable with some oscillations.



The auto-steering device should be able to select the proper control constants,  $k_1$  and  $k_2$ .

It is evident that the dynamically unstable ship can be made to be stable directionally with auto-steering control.



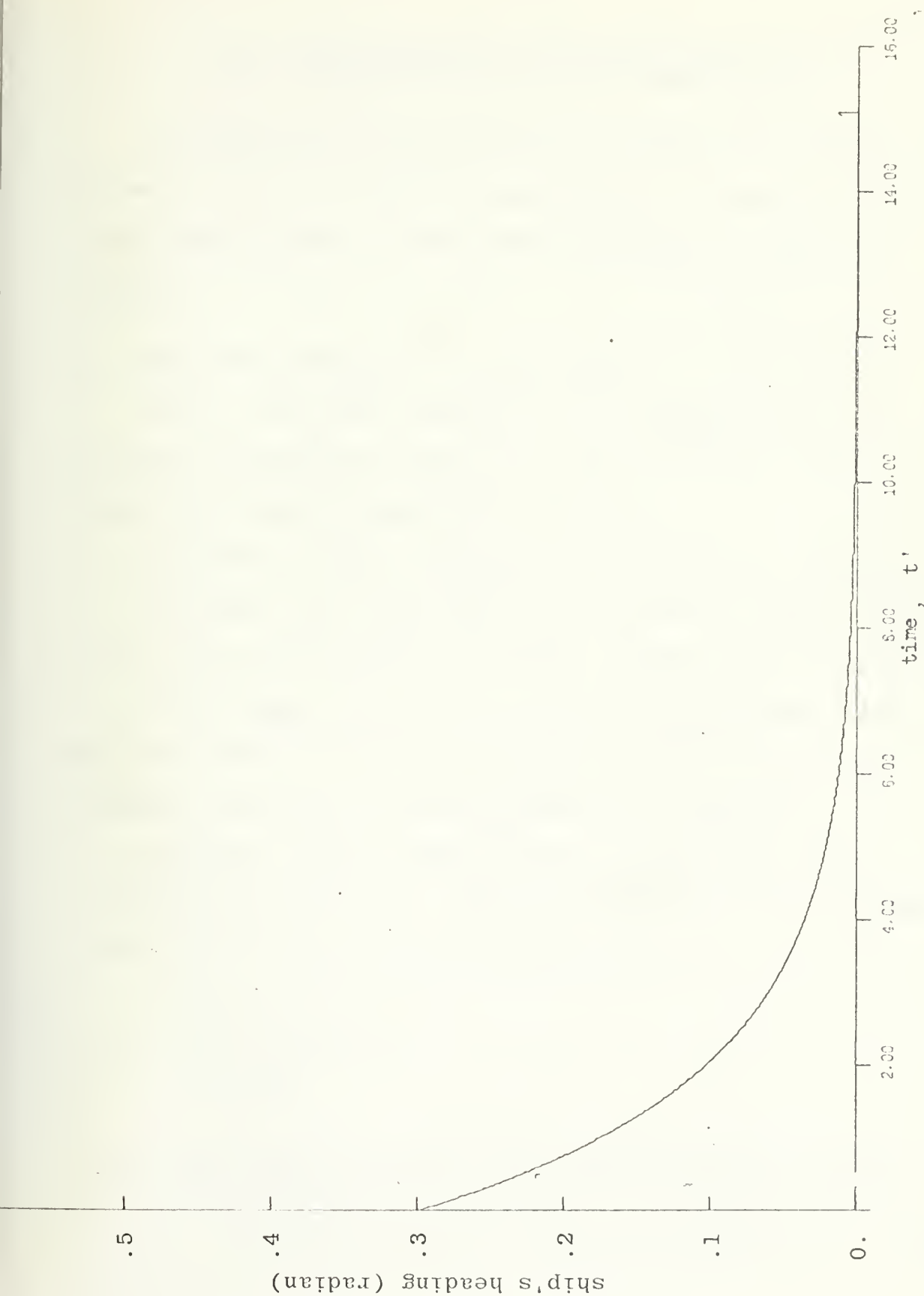


Figure 5-1. Response of ship's heading by auto-steering control after small disturbance.





#### IV. MANEUVERABILITY OF THE UNSTABLE SHIP

To predict the performance of a dynamically unstable ship as well as to get realistic predictions of maneuvers with large rudder angles, a non linear mathematical model is needed.

##### A. NON-LINEAR EQUATION OF SHIP MOTION.

In the non-linear analysis of the ship motion, the following assumptions which are rather typical in the literatures Refs. 2 and 8 were applied.

1) Terms of higher order than third order are neglected.

2) Rudder force and moment derivatives of higher than first order, and effects of rudder angular rate are negligible.

3) Odd terms in  $v$ ,  $r$ ,  $\delta$ ,  $\dot{v}$  and  $\dot{r}$  in the X-force equation and even terms in the same parameters of the Y-force and N-moment equations are eliminated due to symmetry.

4) Cross coupling terms between acceleration and velocity are neglected. Taking the Taylor Series expansion of Eq. (11) with the above assumptions:

$$\begin{aligned}\ddot{u}' &= \frac{f_1(u', v', r', \delta)}{m' - X_{\dot{u}}'} \\ \ddot{v}' &= \frac{(I_z' - N_{\dot{r}}') f_2(u', v', r', \delta) - (m' x_G' - Y_{\dot{r}}') f_3(u', v', r', \delta)}{(m' - Y_{\dot{v}}')(I_z' - N_{\dot{r}}') - (m' x_G' - N_{\dot{v}}')(m' x_G' - Y_{\dot{r}}')} \quad (27)\end{aligned}$$



$$\dot{\gamma}' = \frac{(m' - \gamma_{\dot{v}}') f_3(u', v', r', \delta) - (m' \chi_{\dot{r}}' - N_{\dot{v}}') f_2(u', v', r', \delta)}{(m' - \gamma_{\dot{v}}')(I_2' - N_{\dot{r}}') - (m' \chi_{\dot{r}}' - N_{\dot{v}}')(m' \chi_{\dot{r}}' - \gamma_{\dot{r}}')}$$

where

$$\begin{aligned} f_1(u', v', r', \delta) &= X_0' + X_u' \Delta u' + \frac{1}{2} X_{uu}' \Delta u'^2 + \frac{1}{6} X_{uuu}' \Delta u'^3 + \\ &\quad \frac{1}{2} X_{vv}' v'^2 + (\frac{1}{2} X_{rr}' + m' \chi_{\dot{r}}') r'^2 + \frac{1}{2} X_{\delta\delta}' \delta^2 + \\ &\quad \frac{1}{2} X_{vvu}' v'^2 \Delta u' + \frac{1}{2} X_{rru}' r'^2 \Delta u' + \frac{1}{2} X_{\delta\delta u}' \delta^2 \Delta u' + \\ &\quad (X_{vr}' + m') v' r' + X_{v\delta}' v' \delta + X_{r\delta}' r' \delta + X_{vru}' v' r' \Delta u' + \\ &\quad X_{v\delta u}' v' \delta \Delta u' + X_{r\delta u}' r' \delta \Delta u' \\ f_2(u', v', r', \delta) &= \gamma_0' + \gamma_{0u}' \Delta u' + \gamma_{0uu}' \Delta u'^2 + \gamma_v' v' + \frac{1}{6} \gamma_{vvv}' v'^3 + \\ &\quad \frac{1}{2} \gamma_{vrr}' v' r'^2 + \frac{1}{2} \gamma_{v\delta\delta}' v' \delta^2 + \gamma_{vu}' v' \Delta u' + \frac{1}{2} \gamma_{vuu}' v' \Delta u'^2 + \\ &\quad \gamma_{\delta\delta}' \delta + \frac{1}{6} \gamma_{\delta\delta\delta}' \delta^3 + \frac{1}{2} \gamma_{\delta vv}' \delta v'^2 + \frac{1}{2} \gamma_{\delta rr}' \delta r'^2 + \\ &\quad \gamma_{\delta u}' \delta \Delta u' + \frac{1}{2} \gamma_{\delta uu}' \delta \Delta u'^2 + \gamma_{vr\delta}' v' r' \delta \\ f_3(u', v', r', \delta) &= N_0' + N_{0u}' \Delta u' + N_{0uu}' \Delta u'^2 + N_v' v' + \frac{1}{6} N_{vvv}' v'^3 + \\ &\quad \frac{1}{2} N_{vrr}' v' r'^2 + \frac{1}{2} N_{v\delta\delta}' v' \delta^2 + N_{vu}' v' \Delta u' + \frac{1}{2} N_{vuu}' v' \Delta u'^2 + \\ &\quad (N_r' - m' \chi_{\dot{r}}' u') r' + \frac{1}{6} N_{rrr}' r'^3 + \frac{1}{2} N_{rvv}' r' v'^2 + \frac{1}{2} N_{r\delta\delta}' r' \delta^2 + \\ &\quad N_{ru}' r' \Delta u' + \frac{1}{2} N_{ruu}' r' \Delta u'^2 + N_{\delta\delta}' \delta + \frac{1}{6} N_{\delta\delta\delta}' \delta^3 + \\ &\quad \frac{1}{2} N_{\delta vv}' \delta v'^2 + \frac{1}{2} N_{\delta rr}' \delta r'^2 + N_{\delta u}' \delta \Delta u' + \\ &\quad \frac{1}{2} N_{\delta uu}' \delta \Delta u'^2 + N_{vr\delta}' v' r' \delta \end{aligned} \quad (28)$$



Integrating Eq. (27)

$$\begin{aligned}
 u'(t') &= \int_0^{t'} \dot{u}'(t') dt' \\
 v'(t') &= \int_0^{t'} \dot{v}'(t') dt' \\
 r'(t') &= \int_0^{t'} \dot{r}'(t') dt' \\
 \psi(t') &= \int_0^{t'} \dot{\psi}'(t') dt'
 \end{aligned}
 \tag{29}$$

Note: The time domain should be non-dimensional.

So far the reference axes  $x$ ,  $y$  and  $z$  are fixed to the moving ship. But it is convenient to take the reference axes  $x_0$  and  $y_0$  fixed to the earth in order to determine the path of the ship. Adopting the figure from Ref. 1 gives

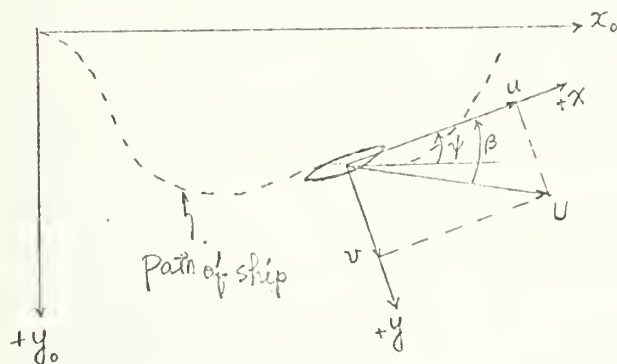


FIGURE 6. Relations between axes fixed in the earth and axes fixed in the ship.



The following relationships may be written as

$$\dot{x}_o(t) = u(t) \cos \psi(t) - v(t) \sin \psi(t) \quad (30)$$

$$\dot{y}_o(t) = u(t) \sin \psi(t) + v(t) \cos \psi(t)$$

$$x_o(t) = \int_0^t \dot{x}_o(t) dt \quad (31)$$

$$y_o(t) = \int_0^t \dot{y}_o(t) dt$$

Non-dimensionalizing Eqs. (30) and (31) using the definitions,

$$x' = \frac{x_o}{L}, \quad \dot{x}' = \frac{\dot{x}_o}{U}, \quad y' = \frac{y_o}{L}, \quad \dot{y}' = \frac{\dot{y}_o}{U}, \quad t' = t \frac{U}{L}$$

and,

$$\frac{x_o}{L} = \int \frac{\dot{x}_o}{L} dt = \int \frac{\dot{x}_o}{U} \frac{U}{L} dt$$

or

$$x'(t') = \int \dot{x}' dt' \quad (32)$$

Likewise,

$$y'(t') = \int \dot{y}' dt'$$





## B. TURNING CIRCLE WITH SELECTED RUDDER DEFLECTION

Fig. 7 shows the typical rudder deflection with respect to time in a turning maneuver.

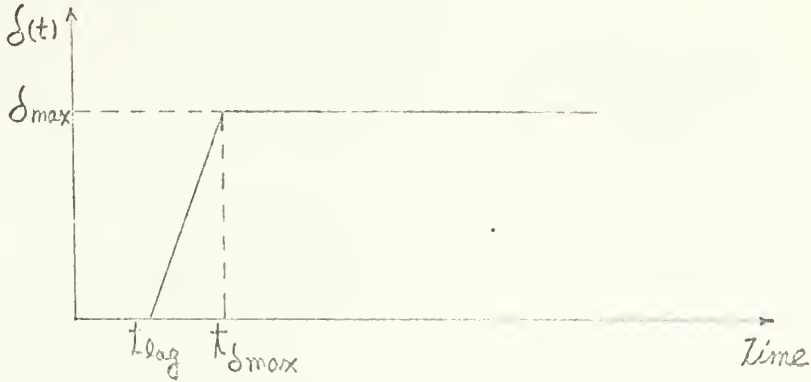


FIGURE 7. Rudder deflection vs.  
time in turning maneuver

Using the ramp function for rudder deflection, then

$$\delta(t) = \alpha [\text{ramp}(t - t_{lag}) - \text{ramp}(t - t_{\delta_{max}})] \quad (33)$$

where

$\delta$  = rudder deflection

$\text{ramp}(t)$  = ramp function

$\alpha$  = rudder deflection rate

$t_{lag}$  = time delay

$t_{\delta_{max}}$  = time at  $\delta_{max}$

Calculation of steady turning radius.

The relation of the angular velocity with linear velocity gives:

$$(\text{radius}) \times (\text{angular velocity}) = (\text{linear velocity})$$



$$\text{or Radius} = \frac{U(t)}{r(t)} = \frac{\sqrt{U^2(t) + V^2(t)}}{r(t)}$$

Taking non-dimensional terms

$$\frac{\text{Radius}}{L} = \frac{U}{UL} \frac{\sqrt{U^2 + V^2}}{r} = \frac{\sqrt{(U/U)^2 + (V/U)^2}}{r(\frac{U}{L})}$$

$$\text{or } Rad' = \frac{\sqrt{(U')^2 + (V')^2}}{L} \quad (34)$$

where  $Rad' = \frac{\text{Radius}}{L}$

Figure 8 shows the typical turning trajectory and terminologies corresponding to Table 5. Eqs. (31) and (34) were simulated for the 5 models using computer program 5 in turning circle maneuvers with a rudder deflection rate of 2 degrees per second for the prototype ( $L = 500'$ ) with zero time lag and constant final angles of  $10^\circ$ ,  $20^\circ$  and  $35^\circ$ . The results are presented in Table 5 and Figs. 8a, 8b and 8c.

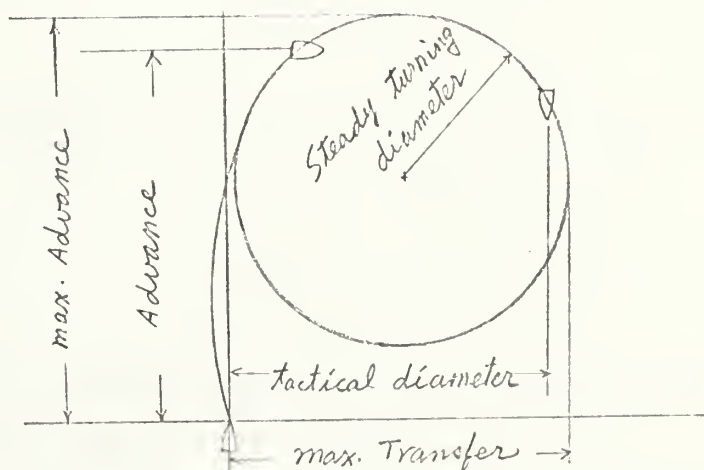


FIGURE 8. Definition of turning circle parameters.



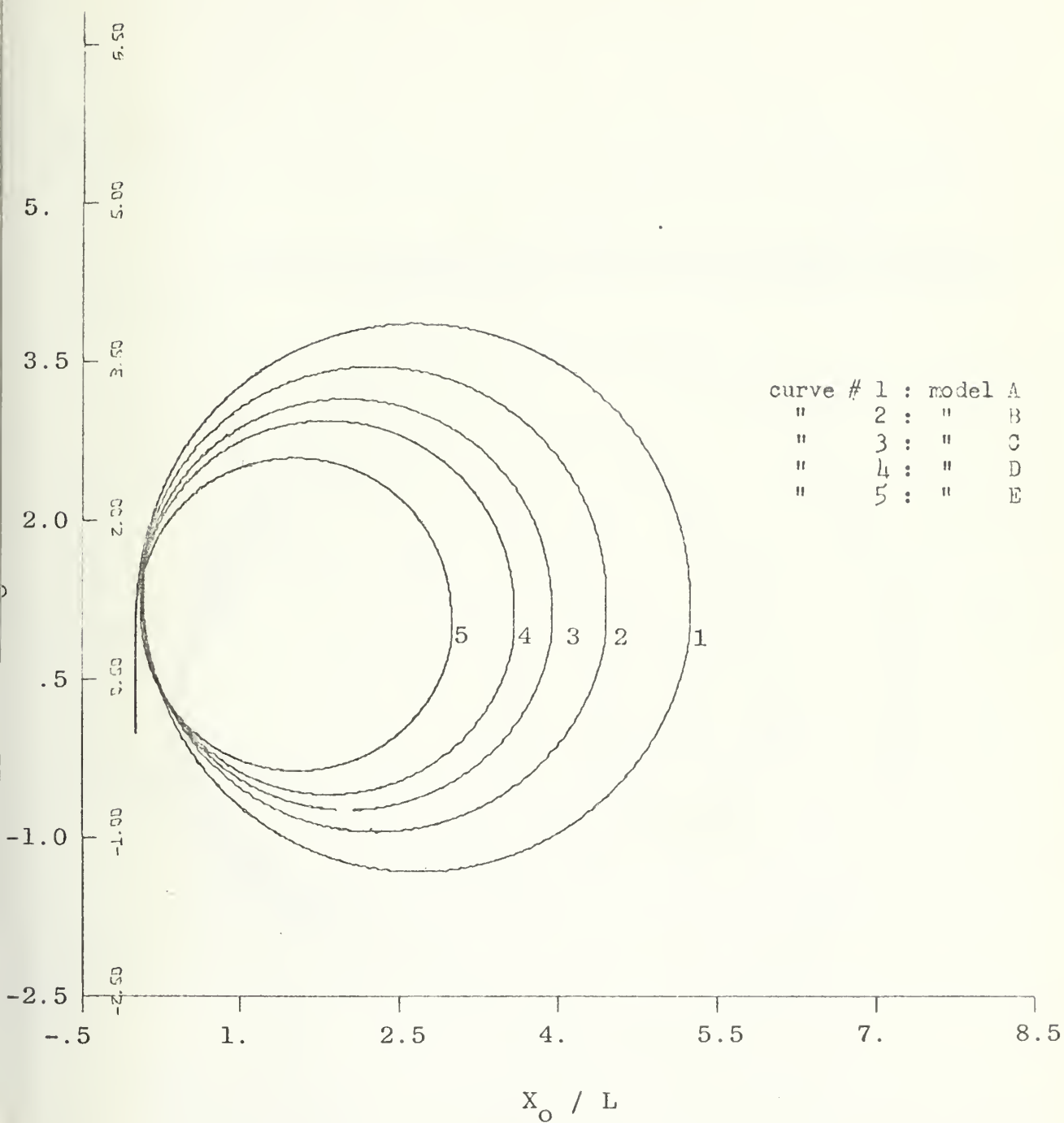


FIGURE 8a. Turning circle of five models with  
 35 degrees of rudder angle.



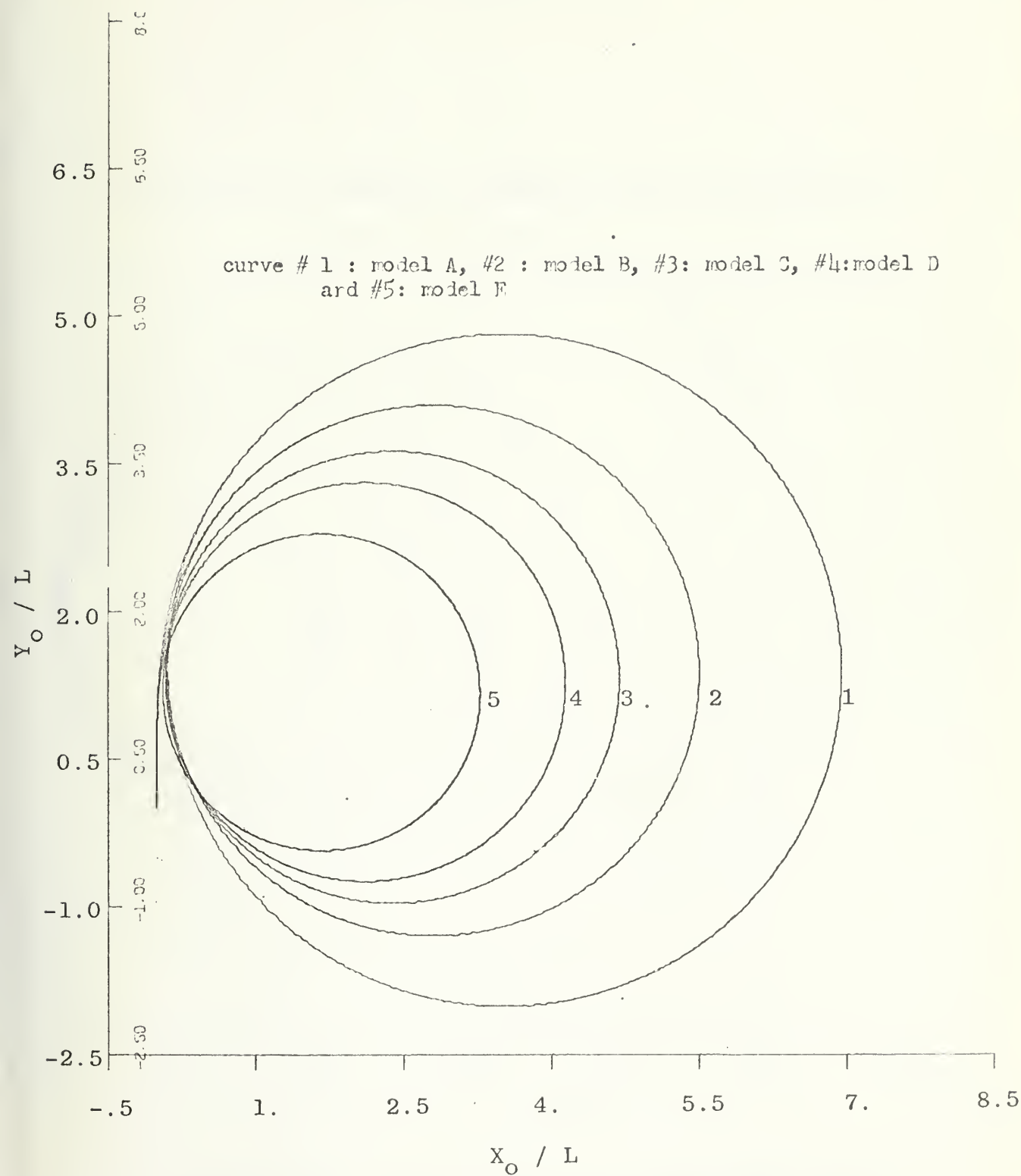


FIGURE 8b. Turning circle of five models with  
25 degrees of rudder angle.





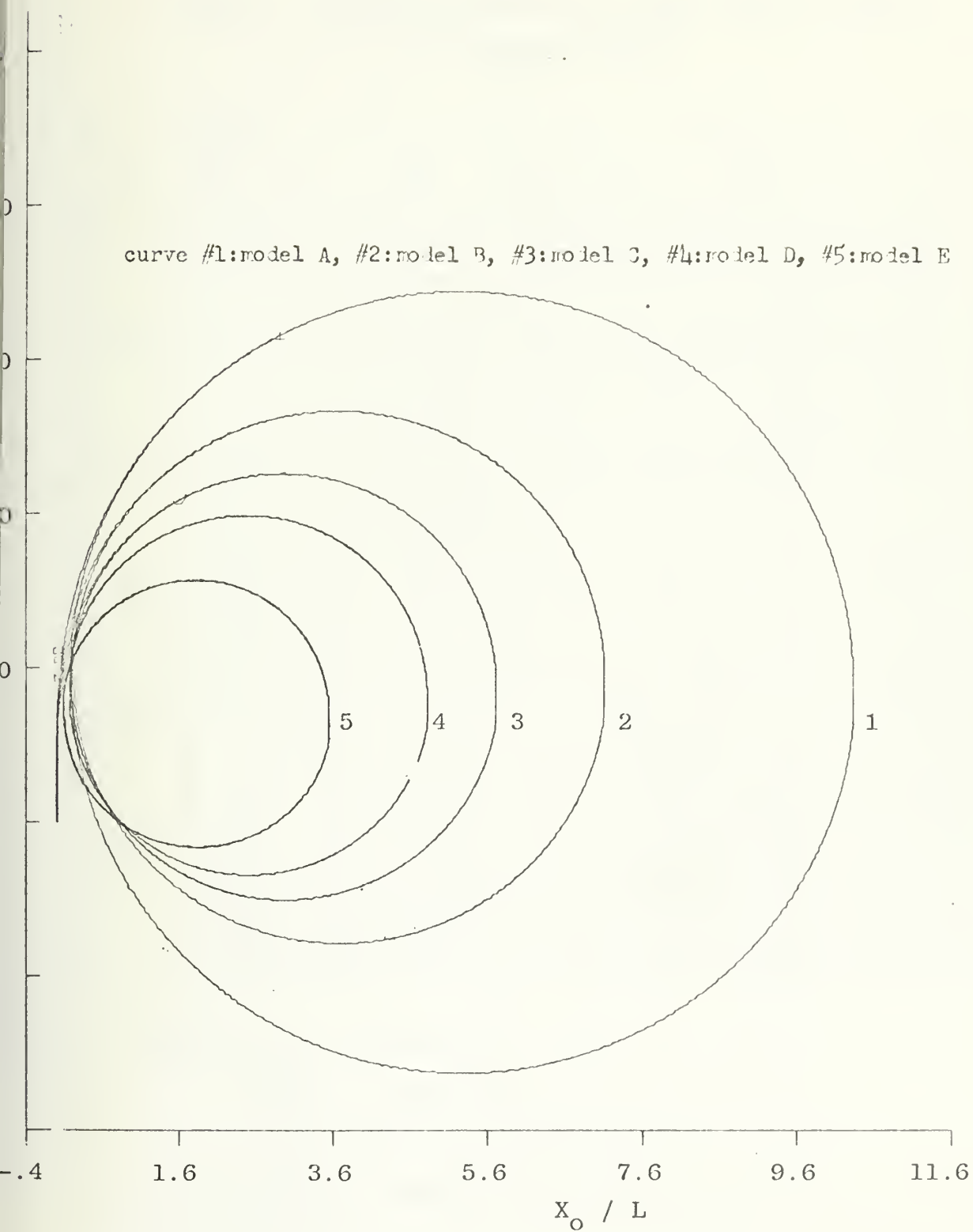


FIGURE 8c. Turning circle of five models with  
10 degrees of rudder angle.



TABLE V. CHARACTERISTICS OF MODEL A,B,C,D AND E  
IN TURNING CIRCLE

$\delta$	Model	Adv.	Max. Adv.	Tran.	T.D.	Max. Tran.	S.T.R.	$t'_{\text{rev}}$
$10^\circ$	A	6.86	6.8825	4.86	10.30	10.322	5.0722	31.6
	B	5.30	5.3224	3.25	7.08	7.1079	3.4498	21.4
	C	4.49	4.5088	2.65	5.88	5.7079	2.7611	17.2
	D	3.94	3.9667	2.18	4.79	4.8122	2.3290	14.6
	E	3.08	3.1277	1.45	3.51	3.5490	1.7295	10.8
$20^\circ$	A	4.80	4.8269	3.16	6.91	6.9411	3.4177	21.2
	B	4.07	4.0996	2.40	5.46	5.4962	2.6946	16.8
	C	3.60	3.6306	2.06	4.66	4.6863	2.2951	14.2
	D	3.29	3.3134	1.80	4.10	4.1379	2.0266	12.6
	E	2.70	2.7892	1.35	3.23	3.2719	1.6070	10.0
$35^\circ$	A	3.840	3.8685	2.33	5.2203	5.2433	2.6364	16.2
	B	3.430	3.4584	1.98	4.435	4.4551	2.1938	13.6
	C	3.120	3.1538	1.77	3.900	3.9380	1.9385	12.0
	D	2.918	2.9475	1.49	3.540	3.5804	1.760	10.8
	E	2.550	2.5943	1.15	2.790	2.9890	1.4727	8.6

Note: Adv. = Advance/L

Tran. = Transfer/L

T.D. = Tactical Diameter/L

S.T.R. = Steady turn radius/L



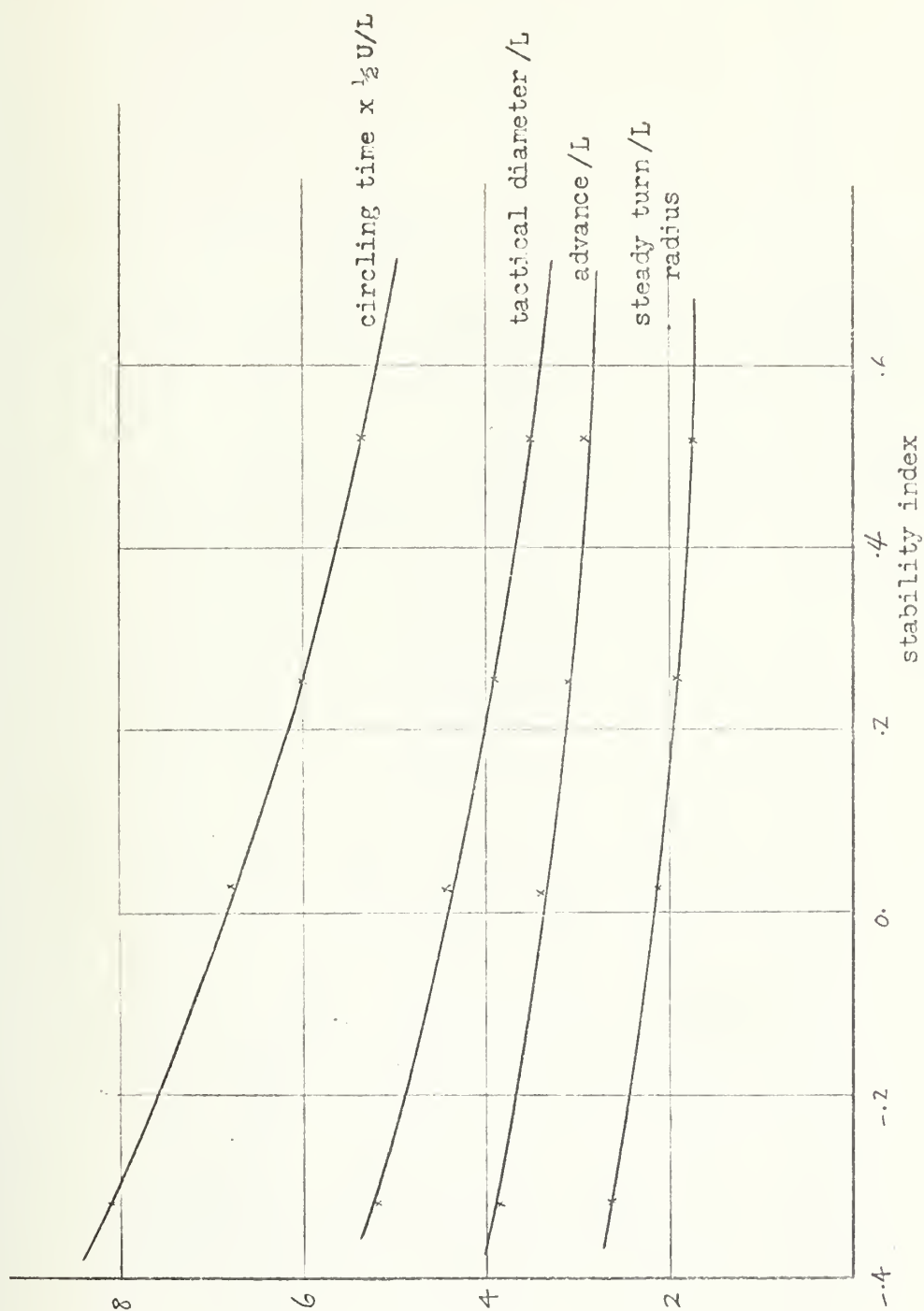


Fig.9 Relations between turning circle parameters and stability index at 35 degree of rudder deflection.



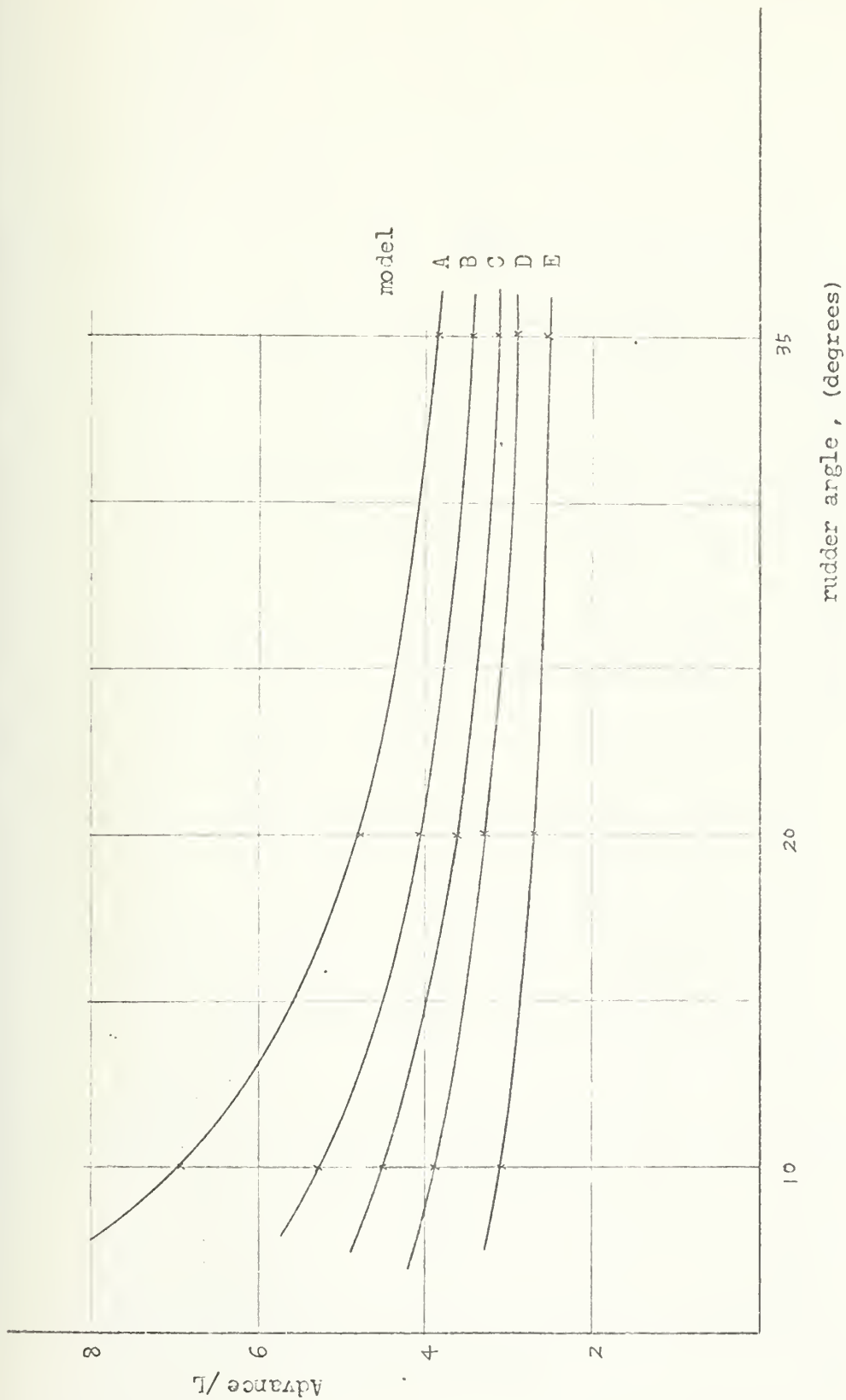


Fig.10 Advance vs. rudder angle in turning circle.





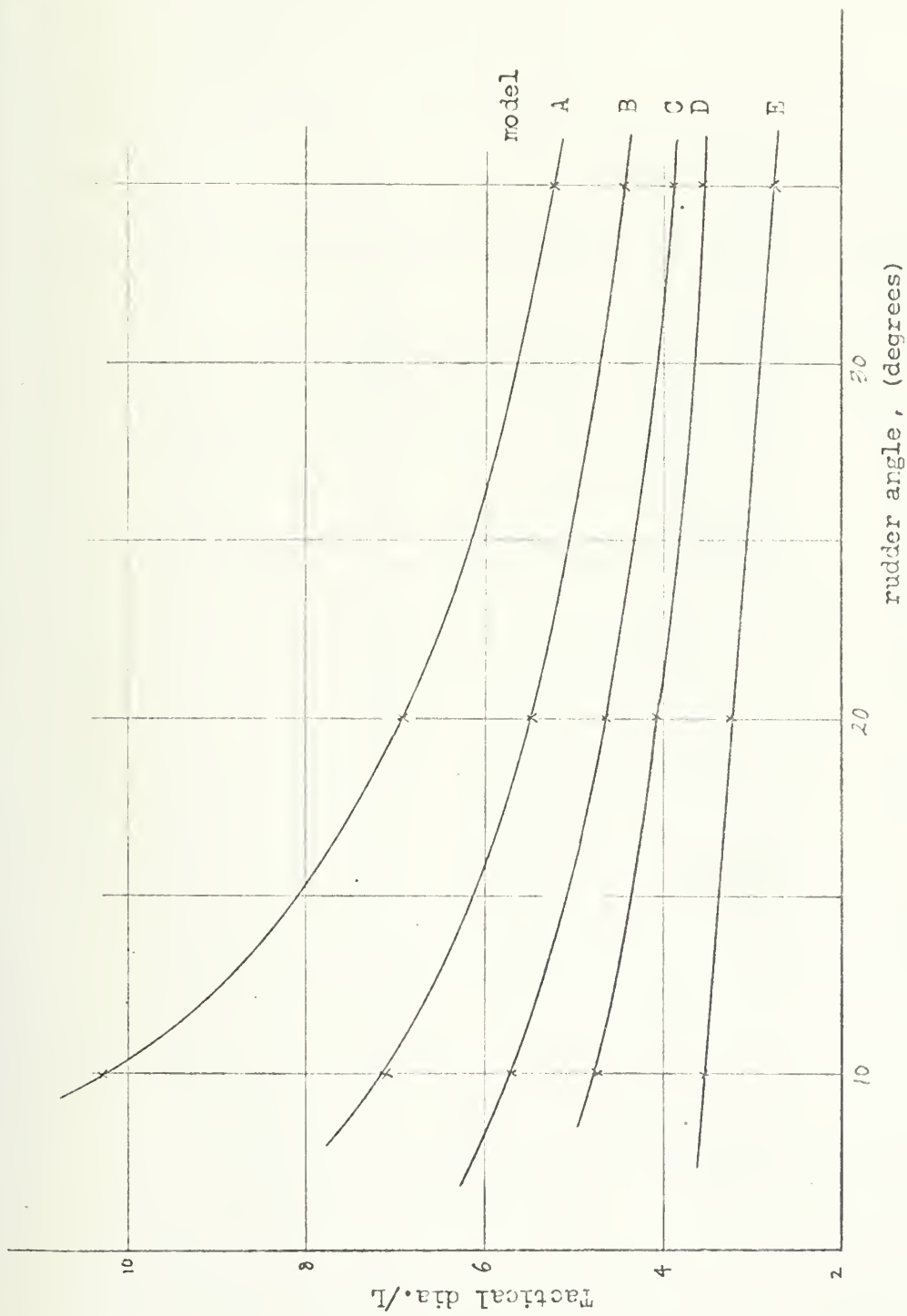


Fig.11 Tactical diameter vs. Rudder angle in turning circle.



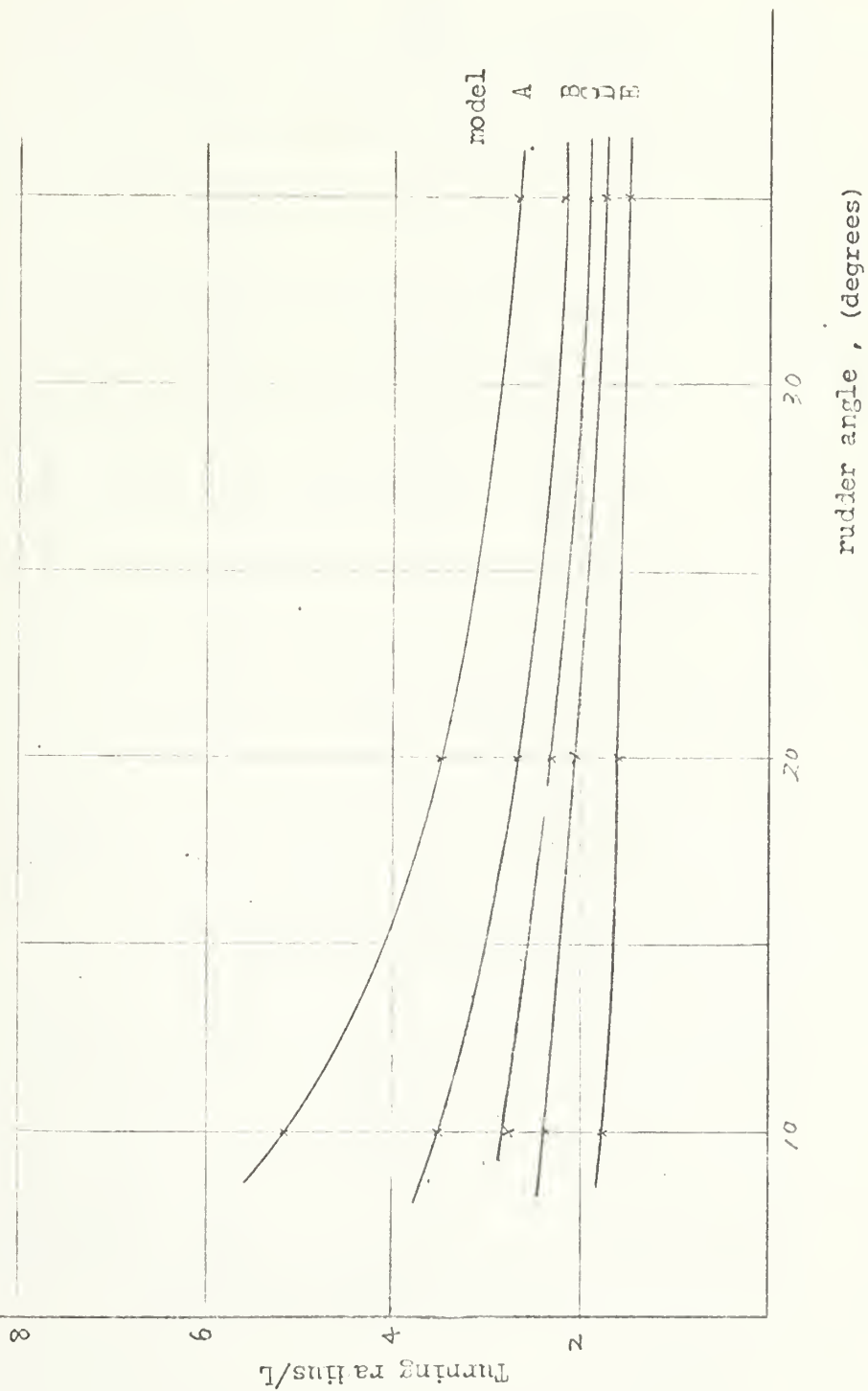


Fig. 12 Steady turning radius vs. rudder angle.



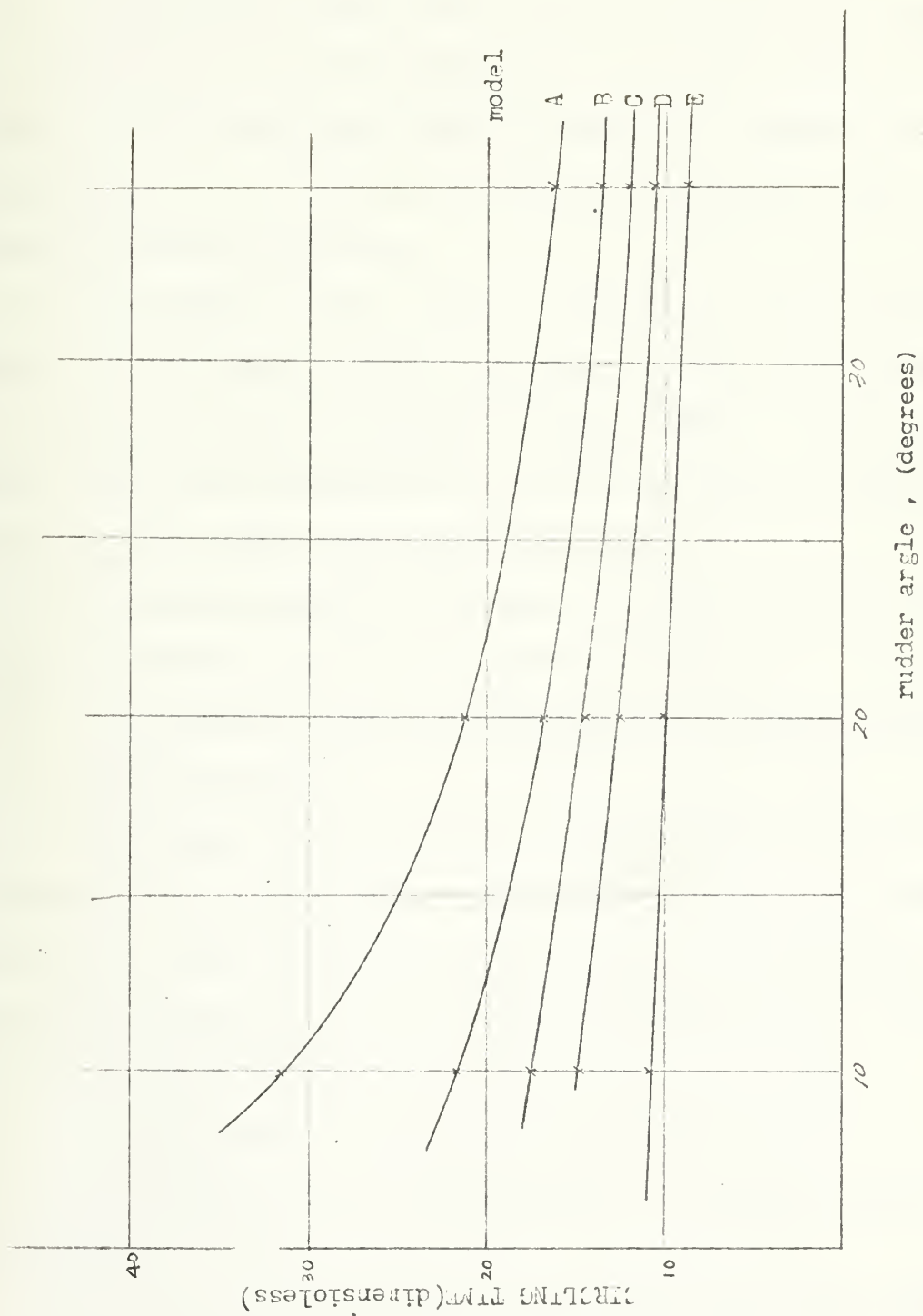


Fig.13 Circling time vs. rudder angle in turning circle.



## Discussion of turning circle maneuver

1) As shown in Fig. 9, the dynamically less stable ship has a smaller tactical diameter, advance, steady turning radius and circling time than the more stable ship, because the yawing rate of the unstable ship is greater than that of stable ship at the same rudder deflection. However, the turning circle parameters are becoming constant as the stability index increases.

2) As shown in Figs. 10, 11, 12 and 13, the rate of change of the turning circle parameters versus the rudder deflection is high at the small rudder angle. In other words, the above parameters are less sensitive to rudder angle as the rudder deflection increases.

3) The more unstable the ship, the less sensitive the above parameters are to the rudder deflection.

### 1. The Effect of the Location of the Rudder on the Turning Circle

A change in the location of the rudder has multiple effects on the ship motion. One is it's effect on dynamical stability, which is the same as the effect of a movable fin described in section (II-C-1) and the other is it's effect on maneuverability.

As described in Appendix A and Ref. 1,

1)  $Y_v$ ,  $\dot{Y}_v$  and  $Y_\delta$  are independent of the position of the rudder.

2)  $N_v$ ,  $Y_r$ ,  $\dot{N}_v$  and  $\dot{Y}_r$  are less positive if the rudder moves forward from the stern.





3)  $N_r$  and  $N_{\dot{r}}$  are less negative if the rudder moves from the stern to the midship position and less positive from midship to the bow.

4) Since the relative flow passing the ship depends on the position considered, the attack angle depends on the location of the rudder. Forward of the pivot point the attack angle is larger than rudder angle, at the pivot point the attack angle is equal to rudder angle and aft of the pivot point the attack angle is smaller than the rudder angle.

5)  $N_{\delta}$  strongly depends on the location of the rudder since  $N_{\delta} = Y_{\delta} \cdot x_f$  where  $x_f$  is the distance of the rudder position from midship and is negative aft of midship.  $N_{\delta}$  is less negative if the rudder moves forward to the bow, but  $N_{\delta \dot{\delta}}$  is less positive if  $\dot{\delta}$  is negative. This less positive effect decreases the yawing rate,  $r$ , due to decreases of  $f_3$  in Eqs. (23) and (28). Decreasing  $r$  in turn causes the turning radius to increase.

On the other hand, the movable fin effect of shifting the rudder forward makes the ship more unstable which makes the turning radius decrease. These two compromising effects are shown in Fig. 14 and 15 which were obtained from computer program 6. In computer program 6, model C was simulated for the evaluation of the effect of the rudder position on turning maneuvers.

As shown on Fig. 14 which represents the effect of the rudder position on the turning radius without replacement of



TABLE VI. THE EFFECT OF RUDDER POSITION ON THE TURNING  
CIRCLE OF MODEL C WITHOUT A FIN AT THE STERN.

$\alpha'_f$	$t'/\text{rev.}$	$v'$	$r'$	$\text{Rad}'$	$\sigma_i$
-0.5	7.12	-0.1617	0.4413	2.2960	0.25616
-0.4	6.84	-0.16571	0.45764	2.25	
-0.3	6.72	-0.1701	0.4688	2.164	0.5482
-0.22	6.64	-0.1735	0.4734	2.144	0.6492
-0.2	6.6	-0.1743	0.4739	2.14	
-0.17	6.64	-0.17556	0.4742	2.1412	0.70581
-0.10	6.64	-0.1784	0.424	2.15	
$\emptyset$	6.76	-0.1822	0.4639	2.19	0.8552
+0.1	7.0	-0.1856	0.4485	2.268	0.90969
0.15	7.16	-0.1871	0.4383	2.32	
0.20	7.44	-0.1885	0.4265	2.386	0.9394
0.21	7.43	-0.18869	0.42398	2.4	0.9410
0.22	7.45	-0.1889	0.4214	2.415	0.9424
0.24	6.08	+0.1345	-0.51396	1.963	0.9445
0.25	6.08	+0.1352	-0.5163	1.955	
0.27	6.04	+0.1366	-0.5205	1.94	
0.30	6.0	+0.1385	-0.5260	1.92	0.9456
0.34	5.92	0.14099	-0.53204	1.898	0.9420
0.5	5.8	0.1493	-0.5425	1.804	0.8980



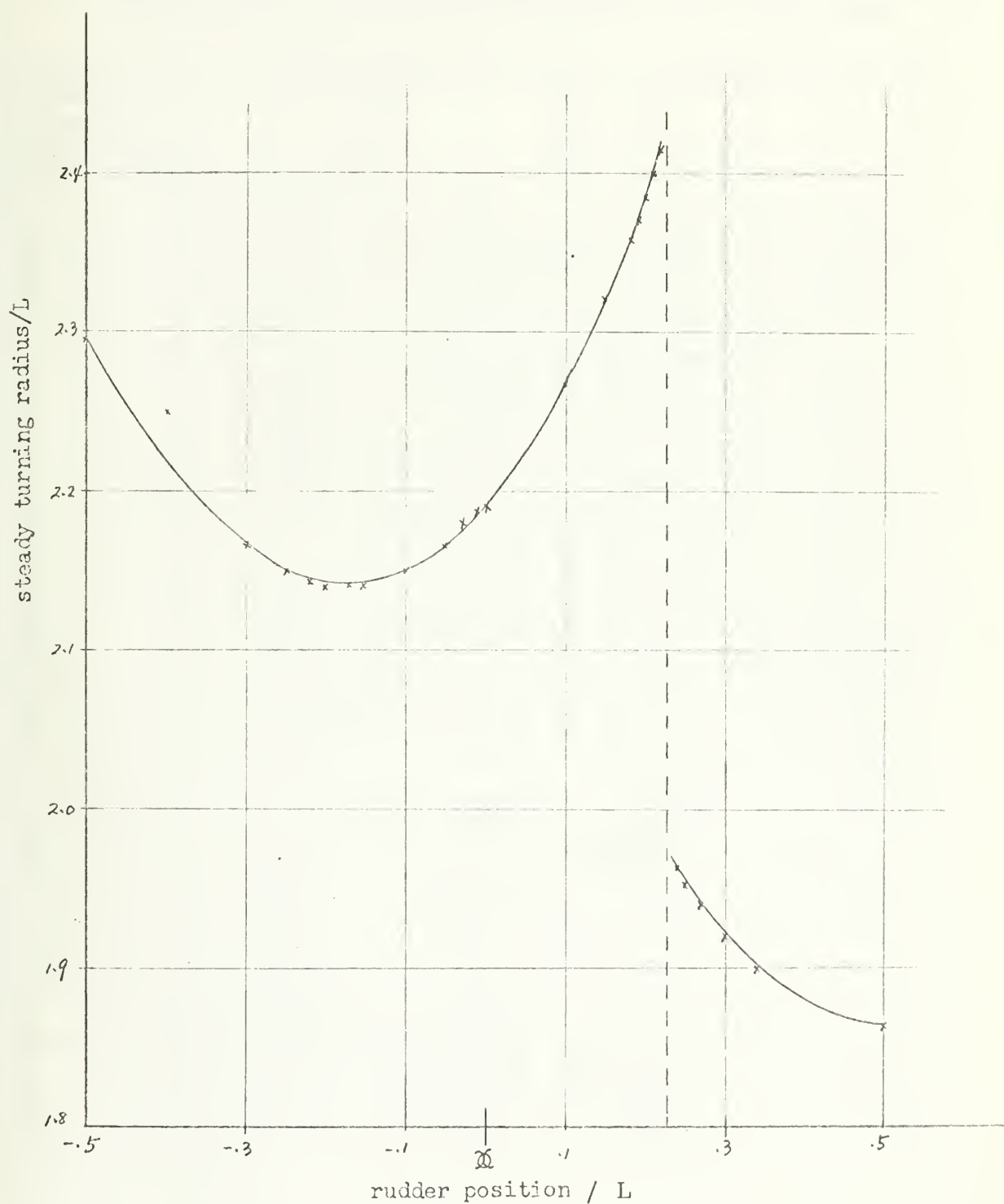


Fig.14 The effect of the position of the rudder on turning radius without a fin at the stern.



TABLE VIa. THE EFFECT OF RUDDER POSITION ON  
THE TURNING CIRCLE WITH A STABILIZING FIN AT THE STERN.

(Rudder Position from $\alpha$ )/L	$v'$	$r'$	$R_{ad}'$	$\sigma_1$
-0.5	-0.1193	0.3053	3.299	-0.3804
-0.4	-0.1233	0.3112	3.238	-0.2570
-0.35	-0.1252	0.3125	3.225	-0.1937
-0.33	-0.1259	0.3127	3.223	-0.1683
-0.3	-0.1269	0.3126	3.225	-0.1305
-0.25	-0.1286	0.3113	3.238	-0.0625
-0.2	-0.13	0.3086	3.268	-0.0088
-0.1	-0.1322	0.2983	3.382	+0.1001
0.	-0.1333	0.2810	3.590	+0.1892
0.2	-0.1292	0.2220	4.542	+0.2948
0.24	-0.1267	0.2054	4.908	+0.3039
0.26	-0.1251	0.1961	5.138	+0.3070
0.27	-0.1242	0.1913	5.268	+0.308
0.28	+0.0572	-0.2827	3.544	+0.309
0.30	+0.0621	-0.2971	3.373	+0.310
0.4	+0.0779	-0.3413	2.939	+0.305
0.5	+0.0874	-0.3639	2.758	+0.2820
	-0.1617	0.4413	2.296	+0.2561





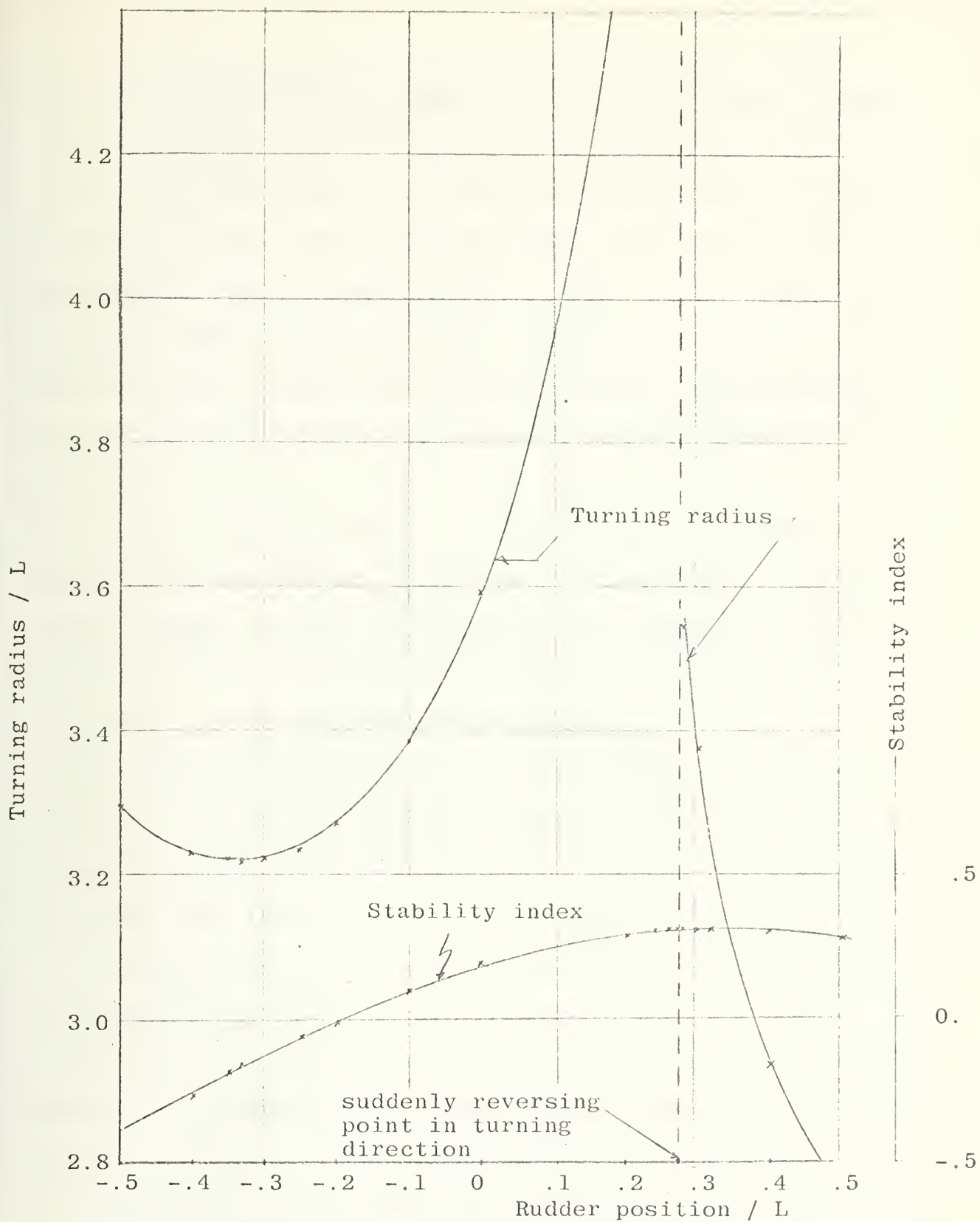


FIGURE 15. The effect of the rudder position on turning circle of model C with a stabilizing fin at the stern.



the fin at the original rudder location, the steady turning radius decreased as the rudder was moved from  $-0.5L$  to  $-0.2L$ , and increased as the rudder was moved from  $-0.2L$  to  $+0.22L$ . At the rudder position  $0.24L$ , the turning radius decreased suddenly from  $2.415L$  to  $1.963L$ . Also, since the turning moment at this point was reversed in direction, the ship suddenly swings to the opposite side. This phenomenon is the characteristic of the unstable ship as described in section IV-D.

If the turning moment is decreased due to shifting the position of the rudder, the yawing rate decreases until the rudder change occurs. After changing the turning direction, the magnitude of yaw rate was suddenly increased. Finally, the turning radius was decreased markedly.

To find the optimum position of the rudder for proper stability and maneuverability of model C, the rudder was moved forward to the bow and in it's location a stabilizing fin was installed.

As shown in Fig. 15 and Table VIa, the turning radius at rudder position  $-0.33L$  is a minimum before a sudden change of rotation and in addition the stability index is negative. Although this minimum turning radius ( $3.223L$ ) is greater than that of the original model C ( $2.296L$ ), it is an acceptable optimum rudder position due to it's better stability index, i.e., negative index.



### C. ZIG-ZAG MANEUVER

According to the standard zig-zag maneuver, the relation of the rudder deflection ( $\delta$ ) and ship's heading ( $\psi$ ) is as described in Fig. 16.

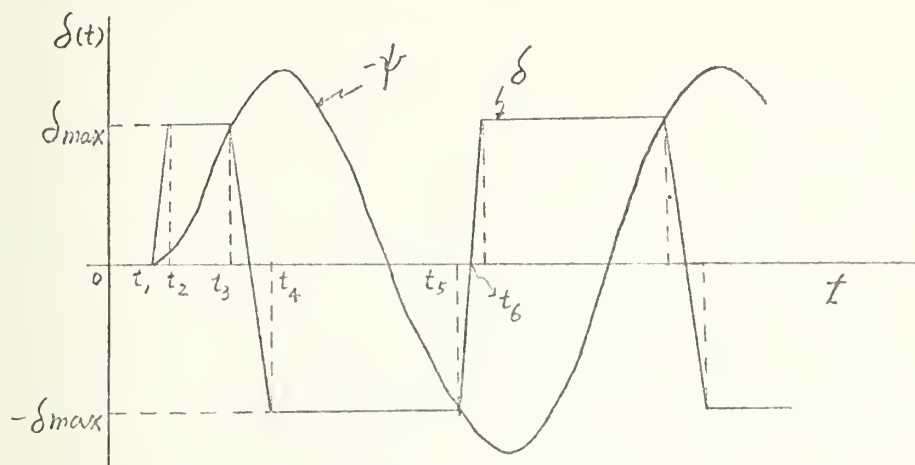


FIGURE 16. Relations between rudder deflection, ship's heading and time in standard zig-zag maneuver.

Using a ramp function for the rudder displacement, the rudder angle is given by:

$$\delta(t) = \alpha [\text{ramp}(t-t_1) - \text{ramp}(t-t_2) - \text{ramp}(t-t_3) + \text{ramp}(t-t_4) + \text{ramp}(t-t_5)]$$

for 1 cycle until  $t \leq t_6$ ,

where

$\alpha$  = rudder rate (degrec/sec.)

$\text{ramp}(t)$  = ramp function

$t_1$  = time delay of rudder

$t_2$  = time at  $\delta = \delta_{\max}$

$t_3$  = time at  $\delta = -\psi$

$t_4$  = time at  $\delta = -\delta_{\max}$

$t_5$  = time at  $\delta = \psi$



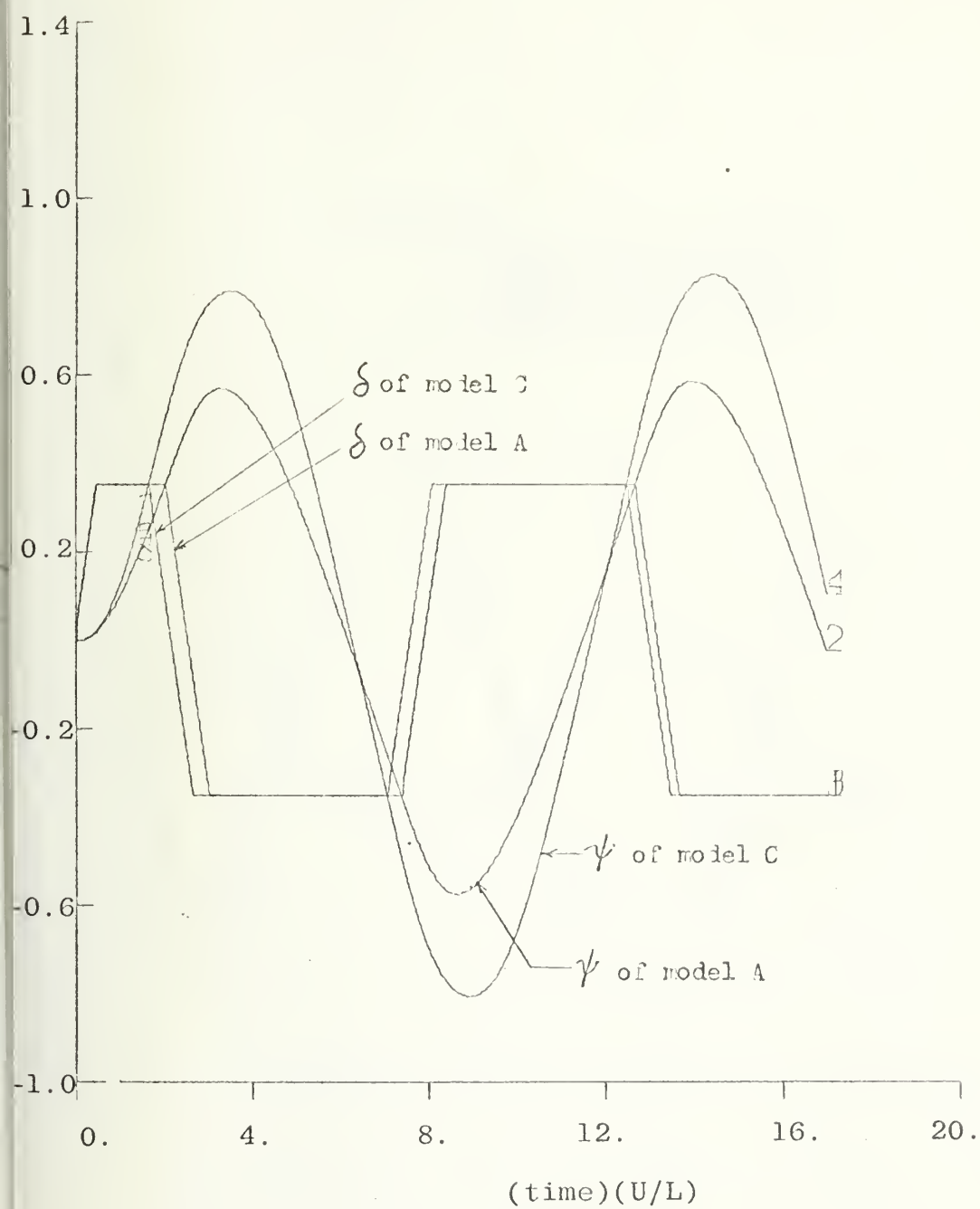


FIGURE 17. Rudder Deflection and Ship's Heading in Z-maneuver.





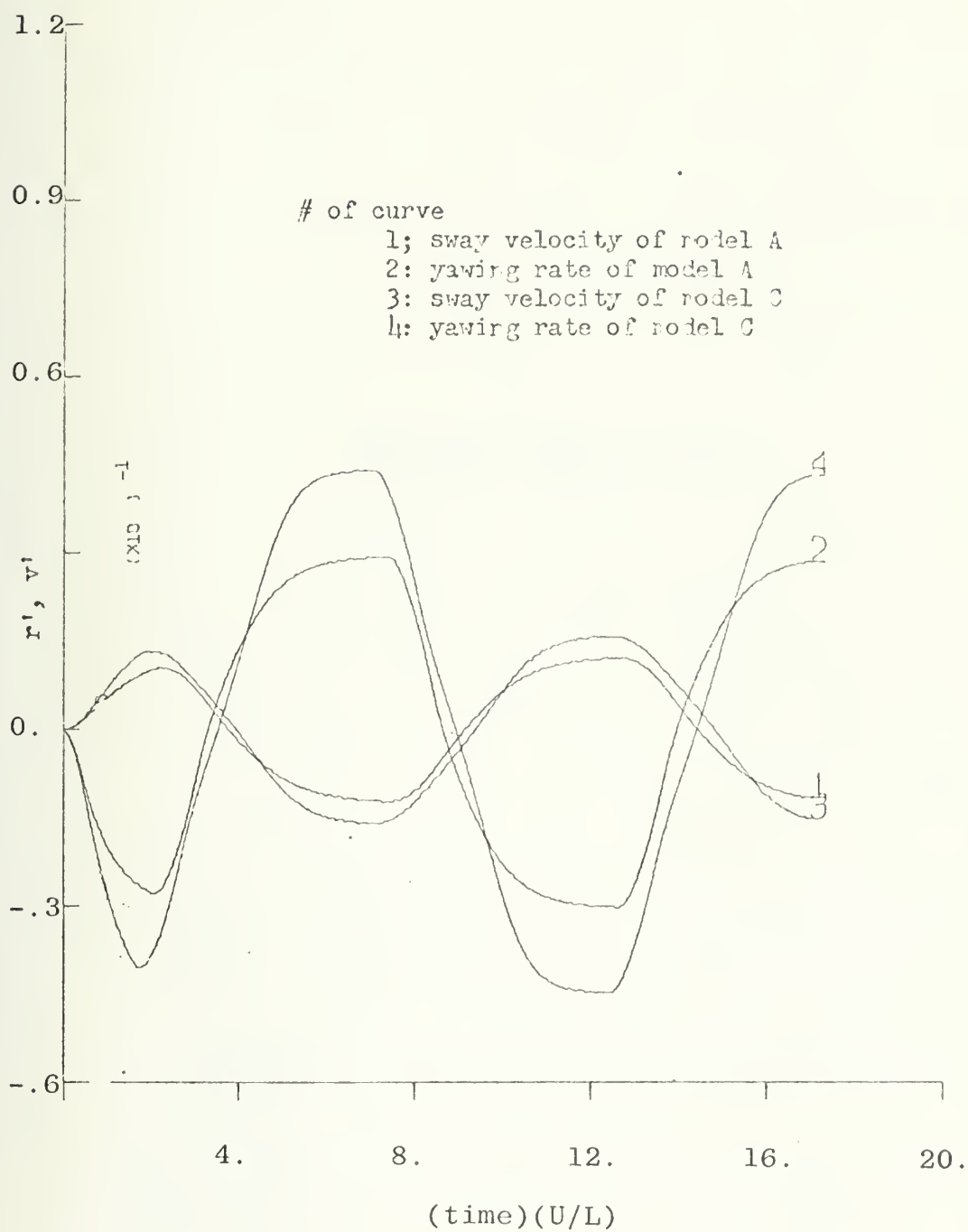


FIGURE 18. Yawing Rate and Sway Velocity in Z-maneuver.



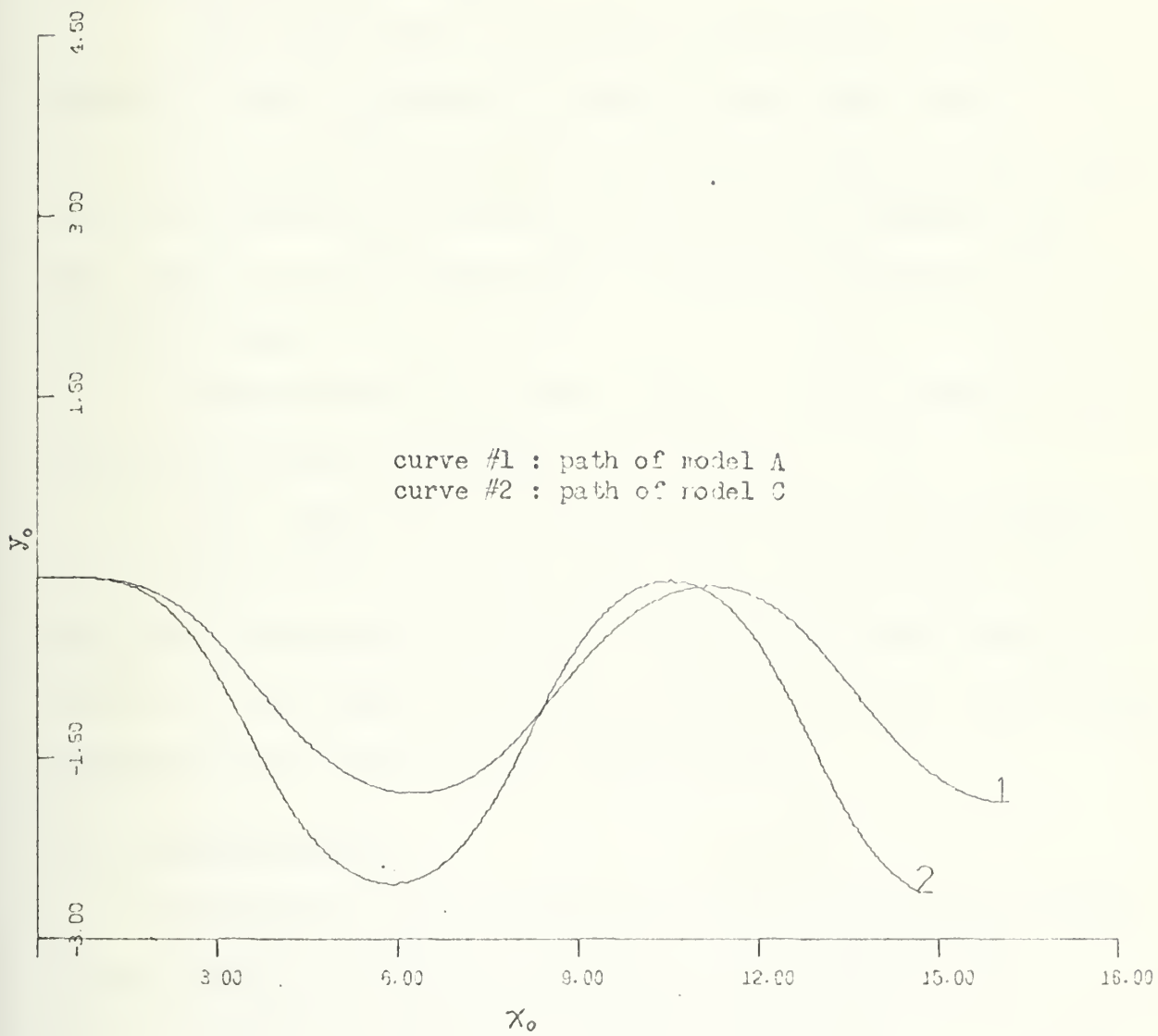


FIGURE 19. Path Trajectory in Z-maneuver.



In computer program 8, the standard zig-zag maneuvers were simulated for model A and C. As shown in Figs. 18, 19 and 20 which represent the results of the program 8, the following can be observed:

1) The amplitude of overshoot in yaw angle of the unstable ship is larger than that of the stable ship.

2) Although the first reversing execution of the rudder of the unstable ship is earlier than for the stable ship, the period of the stable ship is shorter than that of the unstable ship.

3) Amplitude of overshoot of path in the unstable ship is greater than stable ship.

4) The reason for 1), 2) and 3) is that the yaw rate of the unstable ship is greater than that of the stable ship but the sway velocities are almost the same for either ship so that it takes longer for the unstable ship to get the reverse heading after reversing the rudder angle.

#### D. SPIRAL MANEUVER

The standard spiral maneuver of model C was carried out using DSL/360 in computer program 9. The process was started with 10 degrees port rudder and 1 degree of deflecting increment and ended with 10 degrees starboard rudder. The process was repeated.

Figure 20 and Table 7 show the results of computer program 9.

During the spiral test of model C, the angular velocity was reduced suddenly from  $r' = 0.19$  to  $r' = 0.28$  at  $\delta = 1.5$



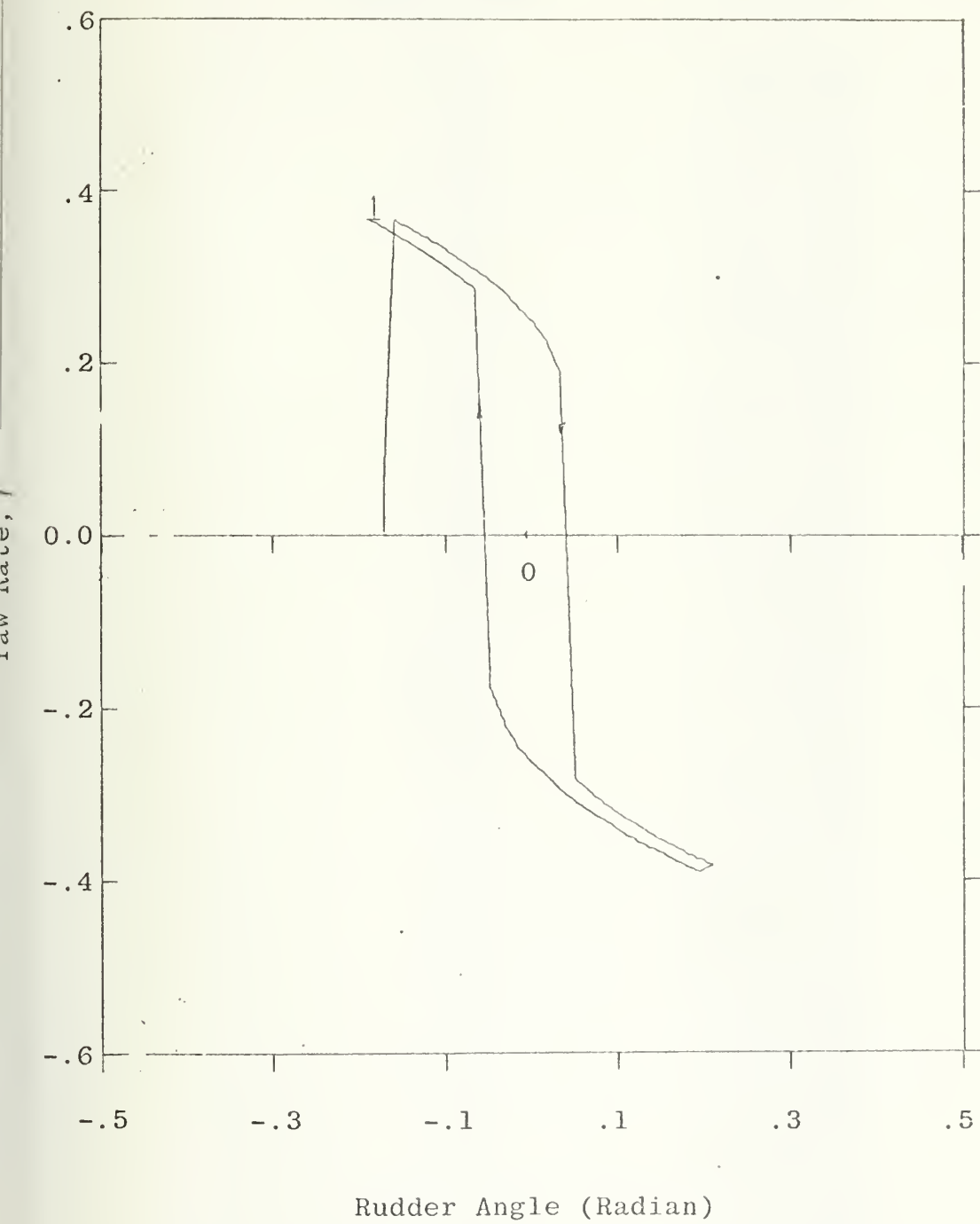


FIGURE 20. Spiral Maneuver of Model C





TABLE VII. SPIRAL MANEUVER OF MODEL C

RUDDER ANGLE (DEGREE)	YAW RATE (RAD./SEC*L/U)
*****	0.36548072
- 9.000	0.35614842
- 8.000	0.34629613
- 7.000	0.33535161
- 6.000	0.32470226
- 5.000	0.31269461
- 4.000	0.29961270
- 3.000	0.28513420
- 2.000	0.26874071
- 1.000	0.24949914
- 0.000	0.22536594
1.000	0.18941998
2.000	-0.28294164
3.000	-0.29785007
4.000	-0.31125194
5.000	-0.32350868
6.000	-0.33485866
7.000	-0.34546858
8.000	-0.35545886
9.000	-0.36492395
10.000	-0.37393415
11.000	-0.38254720
12.000	-0.39080832
11.000	-0.38356121
10.000	-0.37394869
9.000	-0.36493850
8.000	-0.35547382
7.000	-0.34548380
6.000	-0.33437481
5.000	-0.32352638
4.000	-0.31127167
3.000	-0.29787230
2.000	-0.28296709
1.000	-0.26596129
- 0.000	-0.24573880
- 1.000	-0.21966898
- 2.000	-0.17531638
- 3.000	0.28510946
- 4.000	0.29959112
- 5.000	0.31267542
- 6.000	0.32468486
- 7.000	0.33583581
- 8.000	0.34628141
- 9.000	0.35613346
*****	0.36548072



degrees port side. That means any increase of the rudder angle to port by 1.5 degrees will cause the model C to suddenly swing from starboard to port until  $r' = 0.28$ . Likewise, when the rudder angle increase from port to starboard the model C can turn against its rudder by  $\delta = 2.5$  degrees starboard and then suddenly swing in the opposite direction to a new stable position  $r' = 0.28$ .



## V. CONCLUSION

This thesis has presented the characteristics of the dynamically unstable ship in stability and maneuverability.

As discussed in Part III, the most powerful factor in stabilizing a unstable ship is a lifting surface installed aft of midship; the addition of area of fin is more effective for stability than the aspect ratio. The effect of a lifting surface may be obtained by designing a fine stern with neat flow lines and deadwood or a proper rudder. It was found that the fin installed near the pivot point destabilizes the ship.

As discussed in part IV, the unstable ship is more maneuverable in a turning maneuver but less maneuverable in the standard zig-zag maneuver than the stable ship. In turning maneuver, the more unstable ship was less sensitive to the rudder deflection.

As discussed in section IV-B-1, the followings are considered necessary for optimum stability and maneuverability of a ship.

- 1) Locating the rudder at the point aft of the midship where the steady turning radius is minimum.

- 2) Installing a stabilizing fin or deadwood as the stern in order to maintain adequate stability.

DSL/360 Language was found to be very powerful for simulating ship motion.



## VI. RECOMMENDATION

All results in this thesis were obtained by simulation using DSL/360 without real model tests.

The author would like to compare these results from real model tests in the future. The results in the thesis are in non-dimensional values so that they can be converted to full scale ships or models.

Also the following itmes are recommended for future study;

- 1) Maneuverability of the unstable ship in Z-maneuvers other than the standard Z-maneuver.

- 2) Maneuverability of a dynamically unstable ship with auto-steering control.

- 3) Ship design for the optimum stability and maneuverability.





## APPENDIX A

### Prediction of Hydrodynamic Coefficients in Linearized Equation of the Ship.

Using the method in Ref. 1 and 2, the following predictions can be obtained.

(1) A fin travelling at a forward velocity  $u$  and transverse velocity  $v$  has

$$(\gamma'_r)_f = - \left| A'_f \left( \frac{\partial C_L}{\partial \beta} \right)_f \right| \quad (34)$$

where

$$A'_f = A_f / LH$$

$$A_f = \text{Area of fin}$$

$$C_L = \text{Lift Coefficient}$$

$$\beta = \text{Drift angle}$$

$$\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1 + \frac{2}{a_{eff}}} \quad \text{for general aspect ratio fins}$$

$$= \frac{\pi}{2} a_{eff}' \quad \text{for very low aspect ratio fins}$$

$$(N'_r)_f = \chi'_f (\gamma'_r)_f$$

$$(\gamma'_r)_f = \chi'_f (\gamma'_r)_f \quad (34a)$$

$$(N'_r)_f = (\chi'_f)^2 (\gamma'_r)_f$$



$$(\gamma'_v)_f = \frac{2\pi h'_f A'_f}{(a^2 + 1)^{1/2}} \quad (35)$$

where

$a$  = aspect ratio

$a_{eff}$  = effective aspect ratio

$h'_f = h_f/L$

$h_f$  = effective span

$$(N'_v)_f = \chi'_f (\gamma'_v)_f \quad (35a)$$

$$(\gamma'_r)_f = \chi'_f (\gamma'_v)_f$$

$$(N'_r)_f = (\chi'_f)^2 (\gamma'_v)_f$$

Considering the rudder as a fin,

$$\gamma'_\delta = -(\gamma'_v)_f$$

$$N'_\delta = -\frac{1}{2}(\gamma'_\delta) \quad (35b)$$



(2) Considering the ship's hull as a fin with considerable drag coefficient  $C$

$$(\gamma'_v)_h = - \left[ \left( \frac{\partial C_L}{\partial \beta} \right)_h + (C_D)_h \right] \quad (36)$$

where

$$\left( \frac{\partial C_L}{\partial \beta} \right)_h = \frac{\pi}{z} a_{eff} = \frac{\pi H}{L}$$

$$(N'_v)_h = -(m'_2 - k'_1 m') + \chi'_p (\gamma'_v)_h$$

$$(\gamma'_r)_h = -k'_1 m' + \chi'_p (\gamma'_v)_h$$

$$(N'_r)_h = -m'_2 \chi' + \chi'_o (\gamma'_v)_h$$

(36a)

$$(\gamma'_v)_h = -m'_2$$

$$(N'_r)_h = - \frac{k'_1}{\frac{\rho}{z} L^4 H} \int_{bow}^{stern} \frac{L}{H} \left( \frac{\rho}{z} \pi C_S h^2 \chi^2 \right) d\chi$$

$$(\gamma'_r)_h \approx 0$$

$$(N'_i)_h \approx 0$$

(see Ref. 1 for more details)



(3) Assuming a large deadwood which has a sufficiently low aspect ratio, an effective span equal to the draft of the ship and a triangle shape, than

$$(\gamma_v')_{f1} = -\frac{\pi H}{L}$$

$$(N_v')_{f1} = -\frac{1}{2}(\gamma_v')_{f1}$$

$$(\gamma_r')_{f1} = -\frac{1}{2}(\gamma_v')_{f1}$$

$$(N_r')_{f1} = \frac{1}{4}(\gamma_v')_{f1}$$

$$(\gamma_v')_{f1} = \frac{4\pi A_f' h_f'}{(a_{eff} + 1)^{1/2}} \quad (37)$$

$$(N_v')_{f1} = -\frac{1}{2}(\gamma_v')_{f1}$$

$$(\gamma_r')_{f1} = -\frac{1}{2}(\gamma_v')_{f1}$$

$$(N_r')_{f1} = \frac{1}{4}(\gamma_v')_{f1}$$





(4) Assuming that a rudder or small deadwood has

$$\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1 + \frac{2}{a_{eff}}} \quad \text{due to the high aspect ratio, then}$$

$$(\gamma_v')_{f2} = -A'_{f2} \left( \frac{2\pi}{1 + \frac{2}{a_{eff}}} \right)$$

$$(N_v')_{f2} = -\frac{1}{2} (\gamma_v')_{f2}$$

$$(\gamma_r')_{f2} = -\frac{1}{2} (\gamma_v')_{f2}$$

$$(N_r')_{f2} = \frac{1}{4} (\gamma_v')_{f2}$$

(38)

$$(\gamma_{\dot{v}}')_{f2} = -\frac{4\pi \dot{A}'_{f2} h'_{f2}}{(a_{eff} + 1)^{1/2}}$$

$$(N_{\dot{v}}')_{f2} = -\frac{1}{2} (\gamma_{\dot{v}}')_{f2}$$

$$(\gamma_{\dot{r}}')_{f2} = -\frac{1}{2} (\gamma_{\dot{v}}')_{f2}$$

$$(N_{\dot{r}}')_{f2} = \frac{1}{4} (\gamma_{\dot{v}}')_{f2}$$

(see Ref. 1 and 2 for more details)



## APPENDIX B

### Models for Stable, Marginally Unstable, Unstable, very Unstable and Extremely Unstable Ship.

In this section it is intended to obtain models for several types of unstable ships. These will be simulated using DSL/360 to study their stability and maneuverability. The original model was adopted from a five foot Series 60 model (with block coefficient 0.7 and no propeller) as originally developed by the Davidson Laboratory.

#### Principal characteristics of parent model.

Length (L) .....	5.0 ft.
Breadth (B) .....	0.714 ft.
Draft (H) .....	0.267 ft.
Block coefficient ( $C_B$ ).....	0.7
LCG from bow ( $X_G$ ) .....	5.15 ft.
Displacement ( $\Delta$ ).....	41.64 lbs.
Area of rudder ( $A_R$ ).....	0.021 sq. ft.
Rudder span .....	0.2 ft.
Rudder chord .....	0.105 ft.
Mass (m) .....	1.292 slugs
Mass coeff. ( $m'$ ) .....	0.2
Longitudinal added mass	
coeff. ( $k_1 m'$ ) .....	0.004
Lateral added mass coeff. ( $m'_2$ ).....	0.18



Rotational added mass coeff. ( $m'_2$ ).....	0.165
Radius of gyration in air .....	0.25L
C. G. of lateral added mass from midship ( $\frac{\bar{x}}{L}$ ) .....	0.024
Drag coeff. at zero drift angle .....	0.019

By using Appendix A and computer program 1, the parent model can be modified to the desired models. From the results of computer program 1, the five models were selected, which were named stable model A, marginally unstable model B, unstable model C, very unstable model D and extremely unstable model E. (See computer program 1, Appendix A and Table I)

The hydrodynamic derivatives in Table I were considered to be inherent for each model in this thesis.



## APPENDIX C

### Selection of the Optimum Values of the Proportionalities of the Rudder Deflection to the Ship's Heading error and Yawing Velocity.

Let the rudder deflection equation be

$$\delta(t) = k_1 \psi + k_2 \dot{\psi} \quad (\text{see Eq. 23})$$

The ship motion under such rudder control has the characteristic equation,

$$As^3 + Bs^2 + Cs + D = 0 \quad (\text{see Eq. 25})$$

Let

$$s = \frac{B}{A} p, \quad \text{then}$$

$$\frac{B^3}{A^2} p^3 + \frac{B^3}{A^2} p^2 + \frac{CB}{A} p + D = 0$$

$$\text{or} \quad p^3 + p^2 + \frac{AC}{B^2} p + \frac{A^2 D}{B^3} = 0$$

$$\text{or} \quad p^3 + p^2 + b_1 p + b_0 = 0 \quad (39)$$





where

$$b_1 = \frac{AC}{B^2} \quad \text{and} \quad b_0 = \frac{A^2 D}{B^3} \quad (40)$$

From Eqs. (24) and (25)

$$A = (\gamma'_v - m')(N'_r - I'_z) - (\gamma'_r - m'\chi'_G)(N'_v - m'\chi'_G) \quad (41)$$

$$B = c_1 k_2 + c_2$$

where

$$c_1 = N'_G(\gamma'_v - m') - \gamma'_G(N'_v - m'\chi'_G)$$

$$c_2 = (\gamma'_v - m')(N'_r - m'\chi'_G) + \gamma'_v(N'_r - I'_z) - N'_v(\gamma'_r - m'\chi'_G)$$

Likewise

$$C = c_3 k_1 + c_4 k_2 + c_5 \quad (42)$$

where

$$c_3 = N'_G(\gamma'_v - m') - \gamma'_G(N'_v - m'\chi'_G)$$

$$c_4 = N'_G \gamma'_v - N'_v \gamma'_G$$

$$c_5 = \gamma'_v(N'_r - m'\chi'_G) - N'_v(\gamma'_r - m')$$

$$D = c_6 k_1 \quad (43)$$



where

$$c_6 = (Y'_v N'_\delta - N'_v Y'_\delta)$$

Eq. (41) gives

$$B^2 = c_1^2 k_2^2 + 2c_1 c_2 k_2 + c_2^2$$

$$B^3 = c_1^3 k_2^3 + 3c_1^2 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3$$

Eq. (40) gives

$$b_1 = \frac{Ac}{B^2}$$

$$b_1(c_1^2 k_2^2 + 2c_1 c_2 k_2 + c_2^2) = A(c_3 k_1 + c_4 k_2 + c_5) \quad (44)$$

Eq. (40) gives

$$b_o = \frac{A^2 D}{B^3}$$

$$b_o(c_1^3 k_2^3 + 3c_1^2 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3) = A^2 c_6 k_1$$

Thus

$$k_1 = \frac{B_o}{c_6 A^2} (c_1^3 k_2^3 + 3c_1^2 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3) \quad (45)$$

Inserting Eq. (45) into Eq. (44)

$$B_4 k_2^3 + B_3 k_2^2 + B_2 k_2 + B_1 = 0$$

where

$$B_4 = c_1^3 c_3 b_o / c_6 A$$

$$B_3 = \frac{3c_1^2 c_2 c_3 b_o}{c_6 A} - b_1 c_1^2 \quad (46)$$



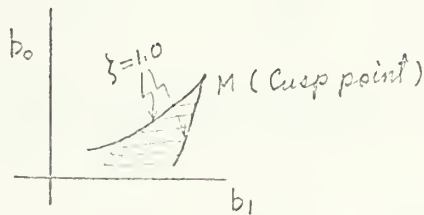
$$B_2 = \frac{3c_1 c_2^2 c_3 b_0}{c_6 A} + A c_4 - 2b_1 c_1 c_2$$

$$B_1 = \frac{c_2^3 c_3 b_0}{c_6 A} + A c_5 - b_1 c_2^2$$

In Eq. (46) all coefficients except  $b_0$  and  $b_1$  can be obtained from hydrodynamic derivatives of a given ship.

Calculator of  $b_0$  and  $b_1$

For 3 real negative roots of the characteristic equation,  $As^3 + Bs^2 + Cs + D = 0$ , the point M on Mitrovics chart (see Ref. 7) should be in  $\zeta = 1.0$  region



For 3 repeated roots, M should be the cusp point.

At the cusp point,

$$\frac{db_0}{db_1} = \text{undefined}$$

$$\frac{db_0}{db_1} = \frac{db_0/dw_t}{db_1/dw_t}$$

Therefore

$$\frac{db_0}{dw_t} = 0 \quad \text{and} \quad \frac{db_1}{dw_t} = 0$$



For the third order characteristic Eq. with  $\gamma = 1.0$

$$b_0 = \omega_t^2 - 2\omega_t^2$$

$$b_1 = 2\omega_t - 3\omega_t^2$$

$$\frac{db_1}{d\omega_t} = 2 - 6\omega_t = 0$$

$$\text{or } \omega_t = \frac{1}{3}$$

$$\text{So, } b_1 = \frac{1}{3} \quad \text{and} \quad b_0 = \frac{1}{27}$$

(see Ref. 7 for more details)

Note: It is difficult to find the exact cusp point by the digital computer due to the non-integer values of  $b_0$  and  $b_1$ .





```

< COMPUTER PROGRAM #1 >

*** SELECTION OF THE DYNAMICALLY UNSTABLE MODELS ***
* THIS WAS PROGRAMMED TO SELECT THE UNSTABLE MODELS BY MODIFYING ***
* THE 5 FOOT SERIES 60 MODEL OF DAVIDSON LAB. ***
* REFERENCE ; APPENDIX A AND B

//PARK1851 JOB (2595,0286FT,NH22),'PARK',TIME=1
//EXEC DSL4,REGION=150K
//DSL.INPUT DD *

LABEL CALCULATION OF HYDRODYNAMIC DERIVATIVES OF SHIP'S MODEL

INTEGER NUM,NPLOT
INTEG RKSEFX

* MODEL DIMENSIONS
CONST DRAFT=.267,LBP=5.,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
CONST XBAR=.024,XO=.307,ZADMAS=.165,IZ=.0125,XG=-.015,UO=1.
CONST NPLOT=10

* FIN DIMENSIONS
CONST EARO=1.5,AF0=.0 ,HFO=.03

* RUDDER DIMENSIONS
CONST EARR=3.8,AFR=.0 ,HFR=.04

* DEADWOOD DIMENSIONS
CONST EAR1=2.5,AF1=.032,HF1=.0534

CONST XF=-0.5
INITIAL
NUM=0

* CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE BARE HULL
YVH=-3.14*DRAFT/LBP-CDH
NVH=-1.*(YADMAS-K1*MASS)+X*P*YVH
YRH=-K1*MASS+X*P*YVH
NRH=-ZADMAS*XBAR+XO*XO*YVH

```



```

YVDTH=-YADMAS
NVDTH=0.
YRDTH=0.
NRDTH=12-0.0237

```

\* CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE MOVABLE FIN

```

CLBFO=2.*3.14/(1.+2./EARO)
YVFO=-AFO*CLBFO
NVFO=YVFO**XF
YRFO=YVFO**XF
NRFO=YVFO**XF**2
YVDTF0=-4.*3.14*HFO*AFO/(EARO**2+1.)***0.5
NVDTF0=YVDTF0**XF
YRDTFO=YVDTF0**XF
NRDTFO=YVDTF0**XF**2

```

\* CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE RUDDER

```

YVFR=-AFR*2.*3.14/(1.+2./EARR)
NVFR=-0.5*YVFR
YRFR=-0.5*YVFR
NRFR=0.25*YVFR
YVDTFR=-4.*3.14*HFR*AFR/(EARR **2+1.)***0.5
NVDTFR=-0.5*YVDTFR
YRDTFR=-0.5*YVDTFR
NRDTFR=0.25*YVDTFR

```

\* CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR LARGE DEADWOOD INCLUDING RUDDER

```

YVFI=-3.14*DRAFT/LBP
NVFI=-0.5*YVFI
YRFI=-0.5*YVFI
NRFI=0.25*YVFI
YVDTFI=-4.*3.14*HFI*AFI/(EAR1 **2+1.)***0.5
NVDTFI=-0.5*YVDTFI
YRDTFI=-0.5*YVDTFI
NRDTFI=0.25*YVDTFI

```

\*\* CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE BARE HULL WITH RUDDER AND MOVABLE FIN BUT WITHOUT DEADWOOD

```

YV0=YVH+YVFO+YVFR
NV0=NVH+NVFO+NVFR
YR0=YRH+YRFO+YRFR
NR0=NRH+NRFO+NRFR
YVDT0=YVDTH+YVDTF0+YVDTFR

```



```

NVDT0=NVDT1+NVDT1FO+NVDT1FR
YRDT0=YRDT1+YRDT1FO+YRDT1FR
NRDT0=NRDT1+NRDT1FO+NRDT1FR
AO=(MASS-YVDT0)*{(IZ-NRDT0)-(YRDT0-MASS*XG)+(NVDT0-MASS*XG)}
BO=-{(MASS-YVDT0)*{(IZ-NRDT0)-(YRDT0-MASS*XG)+(NVDT0-MASS*XG)}
MASS*XG)+(YRDT0-MASS*XG)+(NVDT0-MASS*XG)}
CO=YVO*(NRO-MASS*XG*UO)-NVO*(YRO-MASS*UO)
SGMA01=(-BO/AO+SQRT((BO/AO)**2-4.*CO/AO))*0.5
SGMA02=(-BO/AO-SQRT((BO/AO)**2-4.*CO/AO))*0.5
V01=(SGMA01-SGMA02)/AO
V02=-V01
WRITE(6,10) YVO,NVO,YRO,NRO,YVDT0,NVDT0,YRDT0,NRDT0,XF,AFO,AFR,...
SGMA01

```

\* \*  
 CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR  
 THE BARE HULL WITH RUDDER AND MOVABLE FIN WITH DEADWOOD

```

YV1=YVH+YVFO+YVFI
NV1=NVH+NVFO+NVFI
YR1=YRH+YRFO+YRFI
NR1=NRH+NRFO+NRFI
YVDT1=YVDT1H+YVDT1FO+YVDT1FI
NVDT1=NVDT1H+YVDT1FO+YVDT1FI
YRDT1=YRDT1H+YRDT1FO+YRDT1FI
NRDT1=NRDT1H+NRDT1FO+NRDT1FI
A1=(MASS-YVDT1)*{(IZ-NRDT1)-(YRDT1-MASS*XG)+(NVDT1-MASS*XG)}
B1=-{(MASS-YVDT1)*{(IZ-NRDT1)-(YRDT1-MASS*XG)+(NVDT1-MASS*XG)}
MASS*XG)+(YRDT1-MASS*UO)-NV1*(YR1-MASS*UO)
C1=YV1*(NR1-MASS*XG*UO)-NV1*(YR1-MASS*UO)
SGMA11=(-B1/A1+SQRT((B1/A1)**2-4.*C1/A1))*0.5
SGMA12=(-B1/A1-SQRT((B1/A1)**2-4.*C1/A1))*0.5
V11=(SGMA11-SGMA12)/A1
V12=-V11
WRITE(6,10) YV1,NV1,YR1,NR1,YVDT1,NVDT1,YRDT1,NRDT1,XF,AFO,AF1,...
SGMA11

```

10 FORMAT(' ',13F10.5)

DERIVATIVE

V0=V01\*EXP(SGMA01\*TIME)+V02\*EXP(SGMA02\*TIME)

V1=V11\*EXP(SGMA11\*TIME)+V12\*EXP(SGMA12\*TIME)

CONTRL FINITIM=1.0,DELT=0.005,DELS=0.01

PRINT SGMA

SAMPLE

NUM=NUM+1

CALL DRWG(1,1,NUM,TIME,V0)

CALL DRWG(1,2,NUM,TIME,V1)

TERMINAL

CALL ENDRW(NPLOT)

GO TO (1,2,3,4,5,6,7,8,9),NPLOT



```

9 AFR=.016
  GO TO 11
8 AFO=.0225
  GO TO 11
7 XF=-0.3
  GO TO 11
6 XF=0.0
  GO TO 11
5 XF=0.1
  GO TO 11
4 XF=0.2
  GO TO 11
3 XF=0.3
  GO TO 11
2 XF=0.4
  GO TO 11
1 XF=0.5
  CALL RERUN
11
END
STOP
//PLOT, FT12F001, DD UNIT=SYSDA, SPACE=(CYL,(3,1)),
//      CCB=(RECFM=FBA, LRECL=37, BLKSIZE=3589)
//PLOT, SYSIN, DD *
SELECTION OF MODEL
D.S.PARK 181115
5      0. 0.625      1      8      8

```





# < COMPUTER PROGRAM #2 >

\*\*\*EVALUATION OF THE EFFECT OF FIN LOCATION AND SIZE \*\*\*

```
//PARK1852 JOB (2595,0286FT,NH22),'PARK',TIME=4
// EXEC DSL4,REGION=150K
//DSL INPUT DD*
LABEL EVALUATION OF THE EFFECT OF FIN LOCATION AND SIZE
INTEGER NUM,NPLOT,I
INTEGER RKSEFX
CONST NPLOT=7
CONST DRAFT=0.267,LBP=5.,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
CONST XBAR=.024,XO=.307,ZADMAS=.165,IZ=.0125,XG=-.015,UO=1.
CONST EARO=1.5,AFO=.0225,HFO=.03
INITIAL
NUM=0
CLBFO=2.*3.14/(1.+2./EARO)
```

```
* VECTOR ARRAY; 1:MODEL A
                  2:MODEL B
                  3:MODEL C
                  4:MODEL D
                  5:MODEL E
```

```
STORAG YVH(5),NVH(5),YRH(5),NRH(5),YVDTH(5),NVDTH(5),YRDTH(5),...
        NRDTH(5),YV(5),NV(5),YR(5),NR(5),YVDT(5),NVDT(5),YRDT(5),...
```

```
TABLE YVH (1-5)=-0.354335,-0.31306,-0.31306,-0.31306,-0.18668,...
        NVH (1-5)=-0.093382,-0.11446,-0.12657,-0.14474,-0.17765,...
        YRH (1-5)=+0.07738,+0.05674,+0.04463,+0.02646,-0.00645,...
        NRH (1-5)=-0.06347,-0.05315,-0.04346,-0.03301,-0.02155,...
        YVDTH(1-5)=-0.18797,-0.18675,-0.18675,-0.18675,-0.18000,...
        NVDTH(1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
        YRDTH(1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
        NRDTH(1-5)=-0.01319,-0.01289,-0.01213,-0.01171,-0.01120
```

```
WRITE(6,2000)
2000 FORMAT(4X,XF,8X,SIGMA1,4X,SIGMA2,4X,SIGMA3,4X,
1,SIGMA4,4X,SIGMA5)
DERIVATIVE
```

\* CALCULATION OF HYDRODYNAMIC DERIVATIVES OF MOVING FIN

```
XF=TIME-0.5
YVFO=-AFO*CLBFO
NVFO=YVFO*XF
YRFO=YVFO*XF
NRFO=YVFO*XF**2
```



```

YVDTFO=-4.*3.14*HF0*AFO/((EARO**2+1.)*0.5
NVDTFO=YVDTFO*XF
YRDTFO=YVDTFO*XF
NRDTFO=YVDTFO*XF**2

```

DYNAMIC

\* CALCULATION OF HYDRODYNAMIC DERIVATIVES OF SHIP WITH FIN

```

DO 1000 I=1,5
YV(I)=YVH(I)+YVFO
NV(I)=NVH(I)+NVFO
YR(I)=YRH(I)+YRFO
NR(I)=NRH(I)+NRFO
YVDT(I)=YVDTH(I)+YVDTFO
NVDT(I)=NVDTH(I)+NVDTFO
YRDT(I)=YRDTH(I)+YRDTFO
NRDT(I)=NRDTH(I)+NRDTFO
A(I)=(MASS-YVDT(I))*((IZ-NRDT(I))-(YRDT(I)-MASS*XG)*(NVDT(I)-...
MASS*XG)
B(I)=-((MASS-YVDT(I))*((NR(I)-MASS*XG*UO)+YV(I))*((IZ-NRDT(I))+...
NV(I))*((YRDT(I)-MASS*UO)*(NVDT(I)-MASS*XG)))
C(I)=YV(I))*((NR(I)-MASS*XG*UO)-NV(I))*((YR(I)-MASS*UO)
SGMA(I)=(-B(I)/A(I))+SQRT((B(I)/A(I))**2-4.*C(I)/A(I)))*0.5
CONTINUE
WRITE(6,3000) XF,SGMA
3000 FORMAT(2X,F5.3,4X,5F10.5)
CONTRL FJNTIM=1.0,DELT=0.01,DELS=0.01
SAMPLE

```

1000

```

CONTINUE
WRITE(6,3000) XF,SGMA
3000 FORMAT(2X,F5.3,4X,5F10.5)
CONTRL FJNTIM=1.0,DELT=0.01,DELS=0.01
SAMPLE

```

```

NUM=NUM+1
CALL DRWG(1,1,NUM,XF,SGMA(1))
CALL DRWG(1,2,NUM,XF,SGMA(2))
CALL DRWG(1,3,NUM,XF,SGMA(3))
CALL DRWG(1,4,NUM,XF,SGMA(4))
CALL DRWG(1,5,NUM,XF,SGMA(5))

```

PRINT XF  
TERMINAL

```

CALL ENDRW(NPLOT)
GO TO (1,2,3,4,5,6),NPLOT

```

6 HF0=0.225  
AF0=0.1265

5 HF0=0.015  
AF0=0.0056

4 HF0=0.045  
AF0=0.0506

3 HF0=0.012  
AF0=0.012

3 EARO=2.



```

HFO=0.0173
AFO=0.0056
GO TO 12
2 EARO=3.
HFO=0.0212
GO TO 12
1 EARO=4.02448
HFO=0.02448
12 CALL RERUN
END
STOP
//PLOT, FT12FG01 DD UNIT=SYSDA, SPACE=(CYL,(3,1)),
//      LCB=(RECFM=FBA, LRECL=37, BLKSIZE=3589)
//PLOT, SYSIN DD
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK
-0.5      0.2      -1.5      0.375      5.      8.      05
D.S.PARK

```



# < COMPUTER PROGRAM #3 >

\*\*\* EVALUATION OF THE EFFECT OF THE CENTER OF GRAVITY \*\*\*  
 \*\*\* OF THE SHIP ON STABILITY \*\*\*

```
//PARK1853 JOB (2595,0286FT,NH22),'PARK',TIME=1
// EXEC DSL4,REGION=150K
//DSL INPUT DD
INTEGER NUM,NPLOT,NOCV
INTEG RKSEFX
CONST NPLOT=1
INITIAL
NUM=0

* HYCRODYNAMIC DERIVATIVES OF MODEL C
CONST YVO=-0.31306,NVO=-0.12657,YRO=0.04463,NRO=-0.04346,.,.,.
YVDT0=-0.18675,NVDT0=0.00243,YRDT0=0.00243,NRDT0=-0.01213
CONST MASS=0.2,IZ=0.18,YD=0.0658,ND=-0.0329,XG=-0.015,U0=1.
DERIVATIVE
XG=-0.4+TIME
AO=(MASS-YVDT0)*((IZ-NRDT0)-(YRDT0-MASS*XG)*(NVDT0-MASS*XG)
BO=-((MASS-YVDT0)*(NRO-MASS*XG*U0)+YVO*(IZ-NRDT0)+NVO*(YRDT0-
MASS*XG)+(YRO-MASS*U0)*(NVDT0-MASS*XG))
CO=YVO*(NRO-MASS*XG*U0)-NVO*(YRO-MASS*U0)
SGMAO1=(-BO/AO-SQRT((BO/AO)**2-4.*CO/AO))*0.5
SGMAO2=(-BO/AO+SQRT((BO/AO)**2-4.*CO/AO))*0.5
SAMPLE
NUM=NUM+1
CALL DRWG(1,1,NUM,XG,SGMAO2)
CTRL FINTIM=8,DELT=0.001,DELS=0.001
PRINT 0.05,XG,YVO,NVO,YRO,NRO,CO,SGMAO2
PREPAR 0.05,XG,SGMAO2
GRAPH XG,SGMAO2
PRPLOT XG,SGMAO2
TERMINAL
CALL ENDRW(NPLOT)
END
STOP
//PLCT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//CCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD
```

-0.5	0.2	-0.4	0.2	D.S.PARK	6.
				5.	





```

**< COMPUTER PROGRAM #4 >
** STABILIZATION OF THE UNSTABLE SHIP BY AUTO-STEERING CONTROL **
* REFERENCE; APPENDIX C ***

//PARK1854 JOB (2595,0286FT,NH22),'PARK',TIME=1
// EXEC DSL4,REGION=150K
//DSL INPUT DD*
LABEL STABILIZATION OF UNSTABLE SHIP BY AUTO STEERING CONTROL
INTEGER NUM,NPLOT,IC,IR,I,J
INTEG RKSEFX
CONST NPLOT=1
INITIAL
  NUM=0
  STCRAG A(4),Q(4),E(4),POL(4),K2(4),B(4),IM(4)
  CONST YVO=-0.31306,NVO=-0.12657,YRO=0.04463,NRO=-0.04346,ND=0.01213
  YVDT0=-1.8675,NVDT0=0.00243,YRDT0=0.00243,NRDT0=-0.015,U0=1.
  IC=4,IR=3,MASS=.2,IZ=.18,YD=.0658,ND=-0.0329,XG=-0.015,U0=1.
  A3=(YVDT0-MASS)*(NRDT0-IZ)-(YRDT0-MASS*XG)*(NVDT0-MASS*XG)
  B1=1./3.
  C1=ND*(YVDT0-MASS)-YD*(NVDT0-MASS*XG)
  C2=(YVDT0-MASS)*(NRO-MASS*XG)+YV0*(NRDT0-IZ)-NVO*(YRDT0-MASS*XG)...
  C3=ND*(YVDT0-MASS)-YD*(NVDT0-MASS*XG)
  C4=YVO*ND-NVO*YD
  C5=YVO*(NRO-MASS*XG)-NVO*(YRO-MASS)
  C6=YVO*ND-NVO*YD
  DO 3000 J=1,40
  WRITE(6,1000) J
  FORMAT(//1X,J=,I2)
  BC=0.037035000-0.0000001*J
  B(1)=C2*C3*B0/(C6*A3)+A3*C5-B1*C2*C2
  B(2)=3*C1*C2*C3*B0/C6/A3+A3*(C4-2*B1*C1*C2
  B(3)=3*C1*C2*C3*B0/C6/A3-B1*C1*C2
  B(4)=C1*C3*B0/C6/A3
  CALL PRQD(B,IC,K2,IM,PCL,IR,IER)
  IF (IM(I).EQ.0.) KK2=K2(I)
  KK1=B0/C6/A3**2*(C1**3*KK2**3+3*C2*C1**2*KK2**2+3*C1*C2**2*KK2...
  +C2**3)
  A(4)=A3
  A(3)=(YVDT0-MASS)*(NRO-MASS*XG*U0+KK2*ND)+YVO*(NRDT0-IZ)-...
  NVO*(YRDT0-MASS*XG)-(YRO-MASS*U0+KK2*YD)*(NVDT0-MASS*XG)
  A(2)=(YVDT0-MASS)*(KK1*ND)+YVO*(NRO-MASS*XG*UC+KK2*ND)-...
  NVO*(YRO-MASS*U0+KK2*YD)-KK1*YD*(NVDT0-MASS*XG)
  A(1)=KK1*YVO*ND-KK1*YD*NVO
  CALL PRQD(A,IC,Q,E,PCL,IR,IER)

```



```

DO 1500 I=1,3
WRITE(6,5000)I,Q(I),I,E(I)
5000 FORMAT(IX,'Q(',I1,')= ',F10.5,10X,'E(',I1,')= ',F10.5)
1500 CONTINUE
IF(ABS(E(2)).LT.0.0010) GO TO 4000
3000 CONTINUE
4000 S1=Q(1)
S2=Q(2)
S3=Q(3)
WRITE(6,2000)KK1,KK2
2000 FCRMAT(//IX,'KK1= ',F10.5,10X,'KK2= ',F10.5)
DERIVATIVE
PSI=0.1*(EXP(S1*TIME)+EXP(S2*TIME)+EXP(S3*TIME))
SAMPLE
NUM=NUM+1
CALL DRWG(1,1,NUM,TIME,PSI)
CONTRL FINTIM=15.,DELT=0.01,DELS=0.02
PRINT 0.1,PSI
TERMINAL
CALL ENDRW(NPLOT)
END
STOP
//PLOT FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
// CC8=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *
0. 2. 0. 0.1 8. D.S.PARK 7.

```

05



< COMPUTER PROGRAM #5 > \*\*\* TURNING TEST FOR COMPARISON OF THE UNSTABLE SHIP WITH STABLE SHIP \*\*\*

\* SELECTED RUDDER ANGLE (DMAX): 10, 20 AND 35 DEGREE  
ANGULAR VELOCITY OF RUDDER=2 DEGREE PER SEC FOR PROTYPE SHIP  
TIME DELAY OF STEERING SYSTEM=0.

//PARK1855 JOB (2595,0286FT,NH22),'PARK',TIME=30  
// EXEC DSL4,REGION=150K  
//DSL INPUT DD\*  
INTEGER NUM,NPLOT  
INTEG RKSEFX  
CONST NPLOT=3

\* HYDRODYNAMIC DERIVATIVES OF MODEL A

CONST YVA=-.35435,NVA=-.09382,YRA=.07738,NRA=-.06347,.01319  
CONST YVDTA=-.18797,NVDTA=.00399,YRDTA=.00399,NRDTA=-.01319

\* HYDRODYNAMIC DERIVATIVES OF MODEL B

CONST YVB=-.31306,NVB=-.11446,YRB=.05674,NRB=-.05315,.01289  
CONST YVDTB=-.18675,NVDTB=.00337,YRDTB=.00337,NRDTB=-.01289

\* HYDRODYNAMIC DERIVATIVES OF MODEL C

CONST YVC=-.31306,NVC=-.12657,YRC=.04463,NRC=-.04346,.01213  
CONST YVDTC=-.18675,NVDTC=.00243,YRDTC=.00243,NRDTC=-.01213

\* HYDRODYNAMIC DERIVATIVES OF MODEL D

CONST YVD=-.31306,NVD=-.14474,YRD=.02646,NRD=-.03801,.01171  
CONST YVDTD=-.18675,NVDTD=.00102,YRDTD=.00102,NRDTD=-.01171

\* HYDRODYNAMIC DERIVATIVES OF MODEL E

CONST YVE=-.18668,NVE=-.17765,YRE=-.00645,NRE=-.02155,....  
CONST YVDE=-.18,NVDE=0,YRDE=0,NRDE=-.0112  
CONST YVV=-.12.9,YRR=-.2766,YRV=-.2,NVNV=-2.31,....  
CONST YVVR=-.612,YRRR=-.606,NRVV=-2.84,YVRR=-2.36  
CONST IZ=.0125,MASS=.2,XG=-.015,YO=.00016,N0=-.0003,.0003,YAXO=0.  
CONST ND=-.03293,YD=.06585,UO=1.,VO=0.,RO=0.,XAXC=0.,YAXO=0.  
CONST DMAX=35.

INITIAL  
NUM=0  
LSHIP=500.



```

LMODEL=5.
USHIP=15.*1.6878
UMODEL=1.5*1.6878
PSIO=0.0
TLAG=0.
TIMAX=TLAG+DMAX
DETB=(MASS-YVDTB)*{(IZ-NRDTA)-(MASS*XC-NVDTA)*{(MASS*XC-YRDTA)}
DETB=(MASS-YVDTB)*{(IZ-NRDTB)-(MASS*XC-NVDTB)*{(MASS*XC-YRDTB)}
DETC=(MASS-YVDTA)*{(IZ-NRDTA)-(MASS*XC-NVDTA)*{(MASS*XC-YRDTA)}
DETD=(MASS-YVDTD)*{(IZ-NRDTD)-(MASS*XC-NVDTD)*{(MASS*XC-YRDTD)}
DETE=(MASS-YVDE)*{(IZ-NRDE)*{(MASS*XC-NVDE)*{(MASS*XC-YRDE)}
DRA=40.
DERIVATIVE
TMODEL=TIME*LMODEL/UMODEL
TSHIP=TIME*LSHIP/USHIP
D--DRA*(RAMP(TLAG)-RAMP(TDMAX/CRATE))/57.3
U=INTGRL(UO,UDOT)
VA=INTGRL(VO,VDOTA)
RA=INTGRL(RO,RDOTA)
PSIA=INTGRL(PSIO,RA)
F2A=Y0+YVA*VA+1./6.*YVRR*RA*VA+1./6.*YVVR*VA*VA*2+(YRA-MASS)*...
F3A=NO+NVA*VA+1./6.*YVRR*RA*VA+1./6.*YVVR*VA*VA*2+(YRA-MASS)*...
NMVDTA=(IZ-NRDTA)*F2A-(MASS*XC-NVDTA)*F3A
NMRTA=(MASS-YVDTA)*F3A-(MASS*XC-NVDTA)*F2A
VDOTA=NMVDTA/DETA
RDOTA=NMRTA/DETA
XAXDTA=U*COS(PSIA)-VA*SIN(PSIA)
YAXDTA=U*SIN(PSIA)+VA*COS(PSIA)
XAXA=INTGRL(XAXO,XAXDTA)
YAXA=INTGRL(YAXO,YAXDTA)
VB=INTGRL(VO,VDOTB)
RB=INTGRL(RO,RDOTB)
PSIB=INTGRL(PSIO,RB)
F2B=Y0+YVB*VB+1./6.*YVRR*RB*VB+1./6.*YVVR*RB*VB*2+(YRB-MASS)*...
F3B=NO+NVB*VB+1./6.*YVRR*RB*VB+1./6.*YVVR*RB*VB*2+(YRB-MASS)*...
NMVDTB=(IZ-NRDTB)*F2B-(MASS*XC-NVDTB)*F3B
NMRTB=NMVDTB/DETB
RDOTB=NMRTB/DETB
XAXDTB=U*COS(PSIB)-VB*SIN(PSIB)
YAXDTB=U*SIN(PSIB)+VB*COS(PSIB)
XAXB=INTGRL(XAXO,XAXDTB)
YAXB=INTGRL(YAXO,YAXDTB)
VC=INTGRL(VO,VDOTC)

```





```

RC=INTGRL(R0,RD0TC)
PSIC=INTGRL(PSIO,RC)
F2C=Y0+YVC*VC+1./6.*YVVR*VC**3+1./2.*YVVR*VC*RC**2+(YRC-MASS)*...
RC+1./6.*YRR*RC**3+.5*YR*VV*RC*VC**2+YD**D
F3C=N0+NVC*VC+1./6.*NVV*VC**3+.5*NVV*VC*RC*VC**2+(NRC-MASS*XG)...
RC+1./6.*NRR*RC**3+NRV*RC*VC**2*.5+ND*D
NMVDTC=(IZ-NRDTIC)*F2C-(MASS*XG-YRDTIC)*F3C
NMVDTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
VD0TC=NMVDTC/DETC
RD0TC=NMVDTC/DETC
XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
YAXDTC=U*SIN(PSIC)+VC*CCS(PSIC)
XAXC=INTGRL(XAX0,XAXDTC)
YAXC=INTGRL(YAX0,YAXDTC)
VD=INTGRL(V0,VD0TD)
RD=INTGRL(R0,RD0TD)
PSID=INTGRL(PSIO,RD)
F2D=Y0+YVD*VD+1./6.*YRR*RD**3+1./2.*YVVR*VD*RD**2+(YRD-MASS)*...
RD+1./6.*YRR*RD**3+.5*YR*VV*RD*VD**2+YD**D
F3D=N0+NVD*VD+1./6.*NVV*VD**3+.5*NVV*VD*RC*VD**2+(NRD-MASS*XG)...
RD+1./6.*NRR*RD**3+NRV*RD*VD**2*.5+ND*D
NMVDTD=(IZ-NRDTD)*F2D-(MASS*XG-YRDTD)*F3D
NMVDTD=(MASS-YVDTD)*F3D-(MASS*XG-NVDTD)*F2D
VD0TD=NMVDTD/DETD
RD0TD=NMVDTD/DETD
XAXDTD=U*COS(PSID)-VD*SIN(PSID)
YAXDTD=U*SIN(PSID)+VD*CCS(PSID)
XAXD=INTGRL(XAX0,XAXDTD)
YAXD=INTGRL(YAX0,YAXDTD)
VE=INTGRL(V0,VD0TE)
RE=INTGRL(R0,RD0TE)
PSIE=INTGRL(PSIO,RE)
F2E=Y0+YVE*VE+1./6.*YRR*RE**3+1./2.*YVVR*VE*RE**2+(YRE-MASS)*...
RD+1./6.*YRR*RE**3+.5*YR*VV*RE*VE**2+YD**D
F3E=N0+NVE*VE+1./6.*NVV*VE**3+.5*NVV*VE*RC*VE**2+(NRE-MASS*XG)...
RD+1./6.*NRR*RE**3+NRV*RE*VE**2*.5+ND*D
NMVDTE=(IZ-NRDTIE)*F2E-(MASS*XG-YRDTIE)*F3E
NMVDTE=(MASS-YVDTE)*F3E-(MASS*XG-NVDTE)*F2E
VD0TE=NMVDTE/DETE
RD0TE=NMVDTE/DETE
XAXDTE=U*COS(PSIE)-VE*SIN(PSIE)
YAXDTE=U*SIN(PSIE)+VE*CCS(PSIE)
XAXE=INTGRL(XAX0,XAXDTE)
YAXE=INTGRL(YAX0,YAXDTE)

```

SAMPLE

```

NUM=NUM+1
CALL DRWG(1,1,NUM,YAXA,XAXA)
CALL DRWG(1,2,NUM,YAXB,XAXB)

```



```

CALL DRWG(1,3,NUM,YAXC,XAXC)
CALL DRWG(1,4,NUM,YAXD,XAXD)
CALL DRWG(1,5,NUM,YAXE,XAXE)
FINITIM=39,DELT=0.01,DELS=0.02
PRINT 2,XAXA,XAXB,XAXC,XAXE,YAXA,YAXB,YAXC,YAXD,YAXE
TERMINAL
CALL ENDRW(NPLOT)
GO TO(1,2),NPLOT
2 DMAX=20
GO TO 11
1 DMAX=10
11 CALL PERUN
END
STOP
//PLOT,FT12F001,DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *

```

-0.5	1.5	-2.5	1.5	D.S.PARK	9.	05
				6.		
-0.5	1.5	-2.5	1.5	D.S.PARK	9.	05
				6.		
-0.4	2.	-4.	2.	D.S.PARK	9.	05
				6.		



\*\*\* EVALUATION OF THE EFFECT OF RUDDER LOCATION ON MANEUVERABILITY \*\*\*

```

//PARK1856 JOB (2595,0286FT,NH22),'PARK',TIME=8
//EXEC DSL4;REGION=150K
//DSL INPUT DD *
INTEGER NUM,NPLOT
INTEG RKSEF
CONST NPLOT=9
CONST YVH=-.31306,NVH=-.12657,YRH=.04463,NRH=-.04346,.,.,.
CONST YVDTH=-.18675,NVDTH=-.00243,YRDTH=-.00243,NRDHC=-.01213
CONST YVVV=-.12.9,YRRR=-.2766,YRVV=-.2,NVVV=-.2.31,.,.,.
CONST NVRR=-.612,NRRR=-.606,NRVV=-.2.84,YVRR=-.2.36
CONST IZ=.0125,MASS=.2,XG=-.015,YO=.00016,NO=-.0003,.,.,.
CONST ND=-.03293,YD=.06585,UO=1.,VO=0.,RO=0.,XAXO=0.,YAXO=0.
CONST EARR=3.8,AFR=0.016,HFR=0.04
CONST XF=0.5
INITIAL
  NUM=0
  XU=-.0253
  XUDOT=0.025
  PSI=0
  LSHIP=500.
  LMODEL=5.
  USHIP=15.*1.6878
  UMODEL=1.5*1.6878
  XAXO=0.
  YAXO=0.
  DMAX=20.
  TLAG=0.
  TCMAX=TLAG+DMAX
  FI=0.
  EARO=EARR
  AFO=AFR
  HFO=HFR
  YVFR=0.
  NVFR=-0.5*YVFR
  YRFR=-0.5*YVFR
  NRFR=0.25*YVFR
  YVDTFR=0.
  NVDTFR=-0.5*YVDTFR
  YRDTFR=-0.5*YVDTFR
  NRDTFR=0.25*YVDTFR
  CLBFO=2.*3.14/(1.+2./EARO)
  NDP=YD*XF
  YVFO=-AFO*CLBFO

```



```

NVFO=YVFO*XF
YVFO=YVFO*XF
NRFO=YVFO*XF*2
YVDTFO=-4.*3.14*HF0*AF0/(EARO*2+1.)**0.5
NVDIFO=YVDTFO**XF
YVDTFO=YVDTFO**XF
YVDTFO=YVDTFO**XF*2
YVC=YVH+YVFO-YVFR
NVC=NVH+NVFO-NVFR
YRC=YRH+YRFO-YRFR
NRC=NRH+NRFO-NRFR
YVDTIC=YVDTH+YVDTFO-YVDTFR
NVDIC=NVDTH+NVDTFO-NVDTFR
YRDTC=YRDTH+YRDTFO-YRDTFR
NRDTC=NRDTH+NRDTFO-NRDTFR
DETIC=(MASS-YVDTC)*{(IZ-NRDTIC)*-(MASS*XC-NVDTC)*-(MASS*XC-YRDTIC)}
DRATE=40.
AC=(MASS-YVDTC)*{(IZ-NRDTIC)*-(YRDTIC-MASS*XC)*-(NVDTC-MASS*XC)}
BC=-((MASS-YVDTC)*{(NRC-MASS*XC*UO)+YVC*{(IZ-NRDTIC)+NVC*{(YRDTIC-...
MASS*XC)}+{(YRC-MASS*UO)*{(NVDTC-MASS*XC)}}
CC=YVC*(NRC-MASS*XC*UO)-NVC*{(YRC-MASS*UO)}
SGMAC1=(-BC/AC+SQRT((BC/AC)**2-4.*CC/AC))*0.5
WRITE(6,1500) XF,SGMAC1
FORMAT(5X, 'XF=', F5.3, 5X, 'SIGMA=', F10.5)
1500
DERIVATIVE
TMODEL=TIME*LMODEL/UMCDEL
TSHIP=TIME*LSHIP/USHIP
D=-DRATE*(RAMP(TLAG)-RAMP(TDMAX/CRATE))/57.3
U=INTGRL(UO,UDOT)
VC=INTGRL(VO,VDOTC)
RC=INTGRL(RO,RDOTC)
PSIC=INTGRL(PSIO,RC)
F2C=Y0+YVC*VC+1./6.*YRRR*RC+1./2.*YVVR*VC*RC*2+(YRC-MASS)*...
F3C=NO+NVC*VC+1./6.*YRRR*RC*3+.5*YRVV*RC*VC*2+YD*
F3C=NO+NVC*VC+1./6.*YRRR*RC*3+.5*YRVV*VC*RC*2+(NRC-MASS*XC)*...
NMVDTIC=(IZ-NRDTIC)*F2C-(MASS*XC-YRDTIC)*F3C
NRDTC=(MASS-YVDTC)*F3C-(MASS*XC-NVDTC)*F2C
VDOTC=NMRDTC/DETC
RDOTC=NMRDTC/DETC
XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
YAXDTC=U*SIN(PSIC)+VC*COS(PSIC)
XAXC=INTGRL(XAXO,XAXDOTC)
YAXC=INTGRL(YAXO,YAXDTC)
SAMPLE
NUM=NUM+1
CALL DRWG(1,1,NUM,YAXC,XAXC)
CTRL FINTIM=29.,DELT=0.01,DELS=0.02

```





```

PRINT .50,XAXC,YAXC,PSIC,VC,RC,TMODEL
TERMINAL
CALL ENDRW(NPLOT)
GO TO(1,2,3,4,5,6,7,8),NPLOT
8 XF=0.38
GO TO 11
7 XF=0.36
GO TO 11
6 XF=0.34
GO TO 11
5 XF=0.32
GO TO 11
4 XF=0.31
GO TO 11
3 XF=0.30
GO TO 11
2 XF=0.28
GO TO 11
1 XF=0.27
11 CALL RERUN
END
STOP
//PLOT, FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
// DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLCT,SYSIN DD *
TEST CF RUDDER POSITION ON THE MODEL A
D.S.PARK 061115

```



```

< COMPUTER PROGRAM #7 >

*** RESPONSES OF THE MODEL A,B,C,D AND E TO A SMALL DISTURBANCE ***
* NOTE: SYMBCLS OF VECTOR ARRAY;
* I=1;MODEL A, 2;MDEL B, 3;MODEL C, 4;MODEL D, 5;MODEL E

//PARK1857 JOB (2595,0286FT,NH22),'PARK',TIME=4
//EXEC DSL4,REGION=150K
//DSL INPUT DD
LABEL THE RESPONSE OF THE MODELS TO SMALL DISTURBANCE
INTEGER NUM,NPLOT,I
INTEG RKSF
CCNST NPLOT=1
CONST XDRFT=1.267,LBP=5,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
CONST XBAR=.024,XO=.307,ZADMAS=.165,IZ=.0125,XG=-.015,UO=1.
STORAG YV(5),NV(5),YR(5),YVDT(5),NVDT(5),YRDT(5),...
TABLE A(5),B(5),C(5),S1(5),S2(5),V(5)
YV (1-5)=-0.35435,-0.31306,-0.31306,-0.31306,-0.18668,...
NV (1-5)=-0.09382,-0.11446,-0.12657,-0.14474,-0.17765,...
YR (1-5)=+0.07738,+0.05674,+0.04463,+0.02646,-0.00645,...
NR (1-5)=-0.06347,-0.05315,-0.04346,-0.03801,-0.02155,...
YVDT (1-5)=-0.18797,-0.18675,-0.18675,-0.18675,-0.18000,...
NVDT (1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
YRDT (1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
NRDT (1-5)=-0.01319,-0.01289,-0.01213,-0.01171,-0.01120

INITIAL
NUM=0
WRITE (6,2000)
2000 FORMAT (4X,'TIME',8X,'V1',4X,'V2',4X,'V3',4X,'V4',4X,'V5')
DYNAMIC
DO 1000 I=1,5
A(I)=(MASS-YVDT(I))* (IZ-NRDT(I))-(YRDT(I)-MASS*XG)*(NVDT(I)-...
MASS*XG)
B(I)=-((MASS-YVDT(I))*(NR(I)-MASS*XG*UG)+YV(I)*(IZ-NRDT(I))+...
NV(I)*(YRDT(I)-MASS*XG)+(YR(I)-MASS*UO)*(NVDT(I)-MASS*XG))
C(I)=YV(I)*(NR(I)-MASS*XG*UO)-NV(I)*(YR(I)-MASS*UO)
S1(I)={(-B(I)/A(I))+SQRT((B(I)/A(I))*2-4.*C(I)/A(I))}*0.5
S2(I)={(-B(I)/A(I))-SQRT((B(I)/A(I))*2-4.*C(I)/A(I))}*0.5
V(I)=EXP(S1(I)*TIME)-EXP(S2(I)*TIME)
CONTINUE
1000 WRITE (6,3000) TIME,V
3000 FORMAT (2X,F5.3,4X,F10.5)
CONTRL FINTIME=10.,DELT=0.01,DELS=0.01
SAMPLE NUM=NUM+1
V3=V(3)*0.1

```



```

V4=V(4)*0.005
V5=V(5)*0.000001
CALL DRWG(1,1,NUM,TIME,V(1))
CALL DRWG(1,2,NUM,TIME,V(2))
CALL DRWG(1,3,NUM,TIME,V3)
CALL DRWG(1,4,NUM,TIME,V4)
CALL DRWG(1,5,NUM,TIME,V5)
PRINT S1
TERMINAL CALL ENDRW(NPLOT)
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      CCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *

```

0.	1.5	0.	0.3	D.S.PARK	6.	05
				7.		



< COMPUTER PROGRAM #8 >

# \*\*\*ZIG-ZAG MANEUVERS OF MODEL A AND C\*\*\*

//PARK1858 JOB (2595,0286FT,NH22),'PARK',TIME=8

```
// EXEC DSL4, REGION=150K
```

```
//DSL.INPUT DD *
```

INTEGER NUM,NPLOT

X  
 U  
 S  
 Y  
 R  
 G  
 U  
 T  
 N  
 H

CONST PLOT=1

# ARBITRARY INITIAL TIME FOR RAMP FUNCTION

CONST TA1=0.,TA2=1000.,TA3=1000.,TA4=1000.,TA5=1000.,...

$TB1=0, TB2=1000, TB3=1000, TB4=1000, TB5=1000, \dots$

$$TC1=0, TC2=1000, TC3=1000, TC4=1000, TC5=1000, \dots$$

TD1=0, TD2=1000, TD3=1000, TD4=1000, TD5=1000.

CONST YVA=-.35435, NVA=-.09382, YRA=.07738, NRA=-.06347, ...

```
YVDIA=-.1819,NVDIA=.00399,YKDIA=.00399,NKDIA=-.01319
CONST YVC=-.31306,NVC=-.13257,YBC=-.04443,NBC=-.07246
```

CUNSI YVC=-.31306,NVC=-.12627,YRC=.04463,NRC=-.04246;...  
YVDT=-.18675,NVDT=-.00243,YDTC=-.00243,NDTC=-.01212

```
CONST VVVV=-128 VPPP=-2766 V3VV=-231
YVVIC=-.18813,NVIC=.00243,YKVIC=.00243,NKVIC=-.01213
```

CONS  
1VV=-12.5, 1KK=-.2100, 1KV=-.29KVV=-2.319...  
NVRB=-.612, NRRB=-.600, NRVV=-2.384, VVRB=-2.36

CONST IZ=.0125.MASS=2.XG=-.015.YO=.00016.NQ=-.003....

ND=-.03293,YD=.06585,U0=1.,V0=0.,R0=0.,XAX0=0.,YAX0=0.

INITIAL

$$0 = M \cap N$$

三

$$X_{UDOT}=0.025$$

PSIO=0.0

$$DMAX=20.157.3$$

○  
||  
—  
L

$$\text{DETA} = (\text{MASS} - \text{VDBTA}) * (\text{IZ} - \text{NRDIA}) - (\text{MASS} * \text{XG} - \text{NVDBTA}) * (\text{IZ} - \text{NRDIA}) + (\text{MASS} * \text{XG} - \text{YRDTA}) * (\text{MASS} * \text{XG} - \text{NVDBTA}) + (\text{MASS} * \text{XG} - \text{YRDTA}) * (\text{MASS} * \text{XG} - \text{NVDBTA})$$

DEFINITION 1. Let  $(M, \mathcal{F})$  be a  $\mathcal{F}$ -manifold. A  $\mathcal{F}$ -manifold  $(M, \mathcal{F})$  is called a  $\mathcal{F}$ -manifold with a  $\mathcal{F}$ -manifold structure if  $(M, \mathcal{F})$  is a  $\mathcal{F}$ -manifold and  $(M, \mathcal{F})$  is a  $\mathcal{F}$ -manifold with a  $\mathcal{F}$ -manifold structure.

DERIVATIVE  
DRALE=40.151.3

DERIVATION

$DC = DRATE_{TC1} \cdot (RAMP_{TC1} - RAMP_{TC2}) - RAMP_{TC3} + RAMP_{TC4} + RAMP_{TC5}$   
 $DC = DRATE_{TC1} \cdot (RAMP_{TC1} - RAMP_{TC2}) - RAMP_{TC3} + RAMP_{TC4} + RAMP_{TC5}$   
 $DC = DRATE_{TC1} \cdot (RAMP_{TC1} - RAMP_{TC2}) - RAMP_{TC3} + RAMP_{TC4} + RAMP_{TC5}$

```

DC=DATEF('1C1')-KAMP('1C2')-KAMP('1C3')-KAMP('1C4')-KAMP('1C5')
VA=INTGB1('VC,VDDTA')

```

И  
Т  
С  
М  
О  
А  
У  
В  
З  
Н  
Д  
И  
А

PSIA=INTGR(U)  
=PSIO+U  
=PSIA+U

$$F_2A = Y_0 + Y_1VA + 1.16 \cdot$$
$$RA+1.1/6.4YRRR+RA+3+.5YRVV+RA+VA+2+YD+DA$$
$$F3A = NO + NVA \div VA + 1 \div 6, 5 \times NVVV \div VA \times 3 + 0.5 \times NVRR \div VA \times RA \div 2 \div (NRA - MASS \times XG) \dots$$
$$*RA+I.16, *NRRRA+3+NRVV+RA+VA+2, 5+ND+DA$$

NMVDTA={IZ-NRDTA}+F2Δ-(MASS\*XC-YRDTA)\*F3A

NMRDTA=(MASS-YVDTA)\*F3A-(MASS\*~~XG~~-NVDTA)\*F2A





```

VDDTA=NMVDTA/DETA
RDDTA=NMRTDTA/DETA
XAXDTA=U*COS(PSIA)-VA*SIN(PSIA)
YAXDTA=U*SIN(PSIA)+VA*COS(PSIA)
XAXA=INTGRL(XAXO,XAXDTA)
YAXA=INTGRL(YAXO,YAXDTA)
VC=INTGRL(VO,RDDTC)
PSIC=INTGRL(RC,PSIO,RC)
F2C=Y0+YVC*VC+1./6.*YRR*VC+1./6.*YVVR*VC+1./2.*YVRR*VC*RC+2*(YRC-MASS)*...
RC+1./6.*YRR*VC+1./6.*YVVR*VC+1./6.*YVRR*VC*RC+2*(YRC-MASS)*...
F3C=NO+NV*VC+1./6.*YRR*VC+1./6.*YVVR*VC+1./6.*YVRR*VC*RC+2*(YRC-MASS)*...
NVDTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
NMRTDC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
VDDTC=NMVDTC/DETC
RDDTC=NMRTDC/DETC
XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
YAXDTC=U*SIN(PSIC)+VC*COS(PSIC)
XAXC=INTGRL(XAXO,XAXDTC)
YAXC=INTGRL(YAXO,YAXDTC)
PSIAA=-PSIA
PSICC=-PSIC
DYNAMIC
IF((TIME,LT,TC3).AND.(ABS(DMAX-DC).LT,0.001)) TC2=TIME
IF((TIME,LT,TC3).AND.(ABS(DMAX-DC).LT,0.001)) TC2=TIME
IF((DC,GT,0.0).AND.(ABS(DC+PSIC).LT,0.001)) TC3=TIME
IF((DC,GT,0.0).AND.(ABS(DC+PSIC).LT,0.001)) TC3=TIME
IF((TIME,LT,TC5).AND.(ABS(DMAX+DC).LT,0.001)) TC4=TIME
IF((TIME,LT,TC5).AND.(ABS(DMAX+DC).LT,0.001)) TC4=TIME
IF((DC,LT,0.0).AND.(ABS(DC+PSIC).LT,0.001)) TC5=TIME
IF((DC,LT,0.0).AND.(ABS(DC+PSIC).LT,0.001)) TC5=TIME
IF((TIME,GT,TA5).AND.(ABS(DC).LT,0.001)) TC1=TIME
IF((TIME,GT,TA5).AND.(ABS(DC).LT,0.001)) TC1=TIME
IF((TA2,LT,TC1).RETURN
IF(TC2=1000.
TC3=1000.
TC4=1000.
TC5=1000.
RETURN
TA2=1000.
TA3=1000.
TA4=1000.
TA5=1000.
1500
SAMPLE
NUM=NUM+1
CALL DRWG(1,1,NUM,TIME,DA)

```



```

CALL DRWG(1,2,NUM,TIME,PSIAA)
CALL DRWG(1,3,NUM,TIME,DC)
CALL DRWG(1,4,NUM,TIME,PSICC)
CALL DRWG(2,1,NUM,TIME,VA)
CALL DRWG(2,2,NUM,TIME,RA)
CALL DRWG(2,3,NUM,TIME,VC)
CALL DRWG(2,4,NUM,TIME,RC)
CALL DRWG(3,1,NUM,XAXA,YAXA)
CALL DRWG(3,2,NUM,XAXC,YAXC)
FININTIM=17,DELT=0.005,DELS=0.02
CONTRL .05,DA,DC,PSIA,PSIC,TA1,TA2,TA3,TA4,TA5
PRINT
TERMINAL
CALL ENDRW(NPLOT)
END
STOP
//PLOT. FI12F001 DD UNIT=SYSDA,SPACE={CYL,(3,1)},
//      LCB={RECFM=FBA,LRECL=37,BLKSIZE=3589}
//PLCT.SYSIN DD

```

0.0	4.	-1.	.4	D.S.PARK	6.	05
				5.		
0.0	4.	-0.6	.3	D.S.PARK	6.	05
				5.		
0.	3.	-3.	1.5	D.S.PARK	5.	05
				6.		



# < COMPUTER PROGRAM #9 >

SPIRAL MANEUVER OF MODEL C

```
//PARK1859 JOB (2595,0286FT,NH22),'PARK',TIME=4
// EXEC DSL4,REGION=150K
//DSL INPUT DD
LABEL SPIRAL MANEUVERING
INTEGER NUM,NPLOT
INTEG RKSEFX
CONST NPLOT=1
CONST YVC=-.31306,NVC=-.12557,YRC=.04463,NRC=-.04346,.,.01213
CONST YVDT=-.18675,NVDT=-.00243,YRDT=-.00243,NRDT=-.01213
CONST YVV=-.12.9,YRR=-.2766,YRV=-.2,NVV=-.2.31,.,.
CONST YVR=-.612,NRR=-.606,NRV=-.2.84,YVR=-.2.36
CONST IZ=.0125,MASS=.2,XG=-.015,YO=.00016,NO=-.0003,.,.
CONST ND=-.03293,YD=.06585,UO=1.,VO=0.,RO=0.,XAXO=0.,YAXO=0.
CONST TT=1000.
INITIAL
NUM=0
RC1=0.00
DE=-10./57.3
DETC=(MASS-YVDTC)*(IZ-NRDTC)-(MASS*XG-NVDTC)*(MASS*XG-YRDTC)
PSIO=0.0
WRITE(6,1500)
1500 FORMAT(51X,'RUDDER ANGLE',4X,'YAW RATE',/,51X,'(DEGREE)',4X,
1,'(RAD./SEC#L/U)')
DERIVATIVE
VC=INTGRL(VO,VDTTC)
RC=INTGRL(RO,RDTTC)
PSIC=INTGRL(PSIO,RC)
F2C=Y0+YVC*VC+1./6.*YVVR*RC*RC+1./2.*YVRR*VC*RC*2+(YRC-MASS)*...
F3C=NO+NVC*VC+1./6.*YVVR*RC*RC+1./2.*YVRR*VC*RC*2+YD*
F3C=NO+NVC*VC+1./6.*YVVR*RC*RC+1./2.*YVRR*VC*RC*2+(NRC-MASS*XG)*...
NMVDTTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
NMRDTTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
VDTTC=NMVDTTC/DETC
RDTTC=NMRDTTC/DETC
DYNAMIC
IF(ABS(RDTTC).GT.0.00001) RETURN
RC1=RC
DEGREE=D*57.3
WRITE(6,1000) DEGREE,RC1
1000 FORMAT(53X,1F6.3,7X,1F15.8)
IF(TIME.GT.TT) GO TO 2000
IF(D.GT.+10/57.3) TT=TIME
```



```

D=D+0.017452
RETURN
2000 D=D-0.017452
PRINT RC
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,D,RC1)
      CONTRL FINT IM=425.,DELT=0.01,DELS=0.25
      TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLOT. FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      CCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT. SYSIN DD *
-0.5      0.2      -0.6      0.2      D.S.PARK      6.
                                     120005

```





## LIST OF REFERENCES

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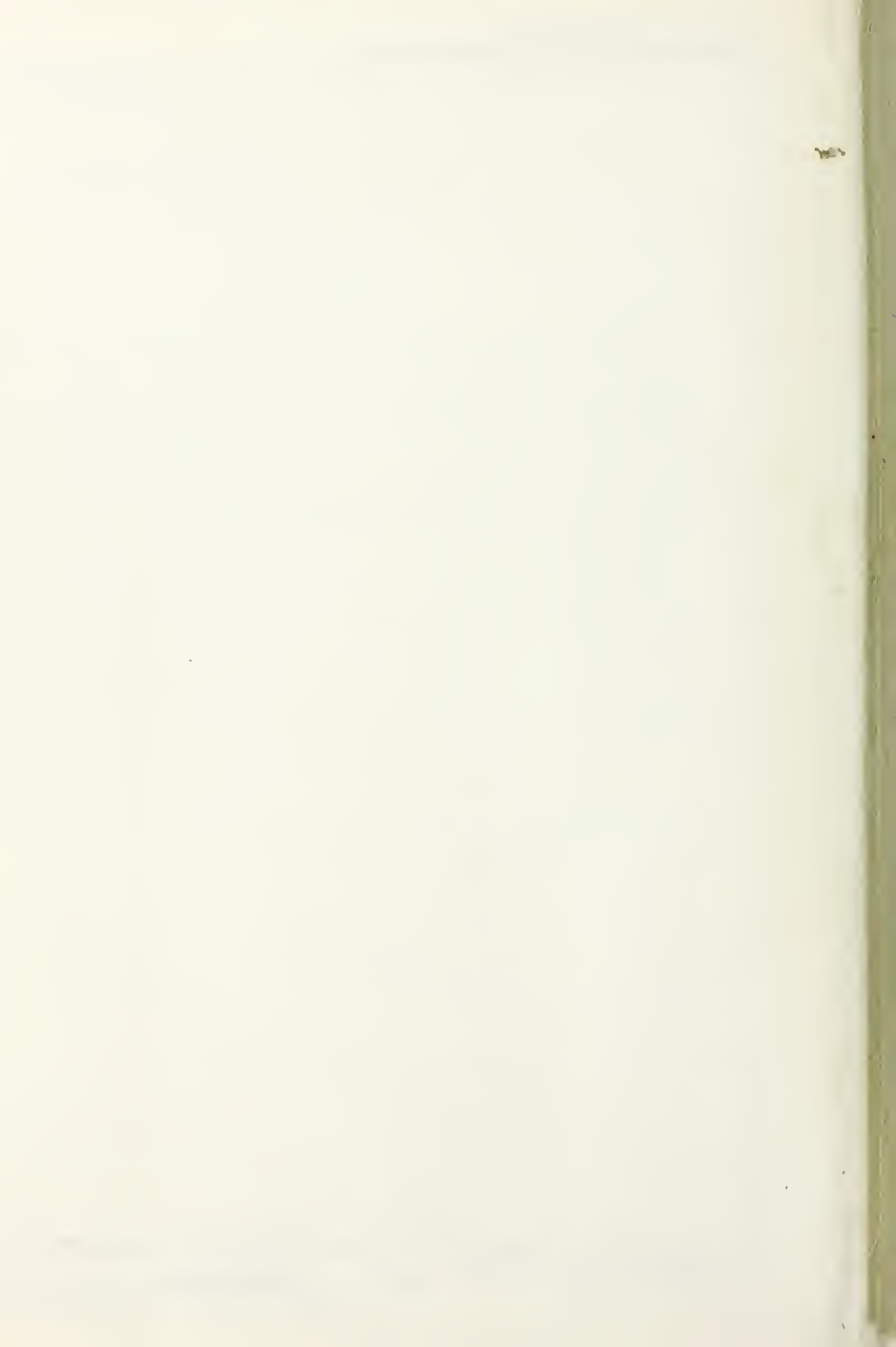
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